INTERNATIONAL RADIO CONSULTATIVE COMMITTEE

C.C.I.R.

DOCUMENTS OF THE

XIth PLENARY ASSEMBLY

OSLO, 1966

REPORTS 413, 414 AND 415

IMPROVED EFFICIENCY IN THE USE OF THE RADIO-FREQUENCY SPECTRUM

Published by the
INTERNATIONAL TELECOMMUNICATION UNION
GENEVA, 1967

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INTRODUCTION

The XIth Plenary Assembly of the C.C.I.R., Oslo, 1966, decided that, in view of the special nature of their contents, Reports 413, 414 and 415 would not be included in Volume III of that Assembly, but published separately.

The text of Resolution 1-1, under which International Working Party III/1 was constituted, is also attached for information.

RESOLUTION 1-1

IMPROVED EFFICIENCY IN THE USE OF THE RADIO-FREQUENCY SPECTRUM

(1963 - 1966)

The C.C.I.R.,

CONSIDERING

(a) that knowledge and technology in the field of radiocommunications are developing rapidly;

(b) that a larger number of simultaneous users of the spectrum must be accommodated in the future;

(c) that the accommodation of these additional users, without serious deterioration of those services in use at present, will require careful consideration of all the many technical factors involved in the simultaneous operation of potentially interfering systems;

(d) that the available information on the wanted-to-interfering signal protection ratios and the operating sensitivities of receiving systems needs further refinement for each of the services, to permit the most efficient planning of the use of the radio-frequency spectrum;

UNANIMOUSLY DECIDES

1. that an International Group of Experts of the C.C.I.R.*, which would be representative of the Study Groups interested in this problem, shall be established, to prepare a report on definitions and procedures for the purpose of enabling the various Study Groups of the C.C.I.R. to provide improved information on:

   – the required signal-to-interference protection ratios,

   – the minimum field strengths required for various classes of emission,

   which would permit the more efficient use of the radio-frequency spectrum by the maximum number of simultaneous users;

2. that the coordination of the work of the Group should be undertaken by Study Group III;

3. that, as far as possible, the work of the Group should be conducted by correspondence.

Note. – The Director, C.C.I.R., is invited to bring this Resolution to the attention of the U.R.S.I. for information.

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* The following Administrations: U.S.A. (Chairman), France, Japan, Netherlands, F.R. of Germany, United Kingdom and the U.S.S.R., together with the E.B.U. and the I.F.R.B., have already indicated their willingness to form part of this Group.

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REPORT 413*

OPERATING NOISE-THRESHOLD OF A RADIO RECEIVING SYSTEM

(Resolution 1-1)

(1966)

1. Introduction

If a receiving system is very sensitive, its operating noise-threshold is low. In the presence of phase interference fading, the operating noise-threshold is determined by that median value of wanted signal power available at the terminals of a loss-free receiving antenna which is required to provide a specified grade of service in the presence of noise, but in the absence of any other unwanted signals. The receiving system includes the receiving antenna in its operating environment, any transmission line to the receiver, and the receiver itself. Radio noise sources, as distinguished from other unwanted signal sources, have spectral energy distributions which vary more or less uniformly with frequency over several decades of the radio-frequency spectrum.

The operating noise-threshold depends on the grade of reception of the wanted signal, and therefore on the kind of service. For example, the quality of a teletype or voice service may depend on the percentage of correctly interpreted received characters. The quality of television may depend on subjective observations which lead to one or more precisely determined grades of reception such as " excellent " or " passable ". In the case of a television service it will be desirable to have separate determinations of the operating noise-thresholds for the sound and vision channels, since these, together with other system considerations, may be expected to lead to an optimum choice of the sound-to-video transmitter power ratio.

In this Report receiving systems which are limited mainly by external noise are referred to as " noise-limited ", and they will in general have much better sensitivities than " gain-limited " systems. In a noise-limited receiving system the gains of the successive stages of the receiver are sufficiently large and the corresponding circuit losses sufficiently low so that the operating noise-threshold is influenced to an appreciable extent by the external noise available from the radiation resistance of the receiving antenna. In a gain-limited receiving system the external noise does not affect the operating noise-threshold appreciably. This Report provides a definition of operating noise-threshold which is applicable to both noise-limited and gain-limited receiving systems. In the particular but very common case of a noise-limited receiving system it is useful to express the operating noise-threshold of the entire receiving system in terms of an operating noise factor for the approximately linear portion of the receiving system, together with an effective noise bandwidth and a required value of operating signal-to-noise ratio at the pre-detection output of the receiving system; much of this document will be concerned with relations of this kind.

The operating threshold of a noise-limited receiving system will depend upon the effective noise bandwidth and on the noises generated in the several components of the receiving system. Both the wanted signal power and the external noise power available from the radiation resistance of the receiving antenna will, in general, depend upon the directivity of the receiving antenna; this is one of the reasons for defining the operating noise-threshold as the wanted signal power at the terminals of an equivalent loss-free receiving antenna, rather than as a required field strength. The most important reason for choosing the reference point at the terminals of an equivalent loss-free receiving antenna is the fact that this is the *only* reference point which provides a proper and unique measure of the sensitivity of the entire receiving system; the argument leading to this conclusion is given in § 10. In practice, measurements are made at accessible terminals and referred to the terminals of an equivalent loss-free antenna. This reference point is, of course, the most natural reference point for separating studies of propagation from studies of receiving systems.

* This Report was adopted unanimously.

When the wanted signal power is subject to phase-interference fading arising from multipath propagation, the operating threshold will depend on the nature and degree of this fading. The operating threshold also depends on the complex spectrum of the fading, i.e. the degree of selective fading within the passband of the receiving system. This latter aspect is beyond the scope of this Report; for digital communication systems the recent work of Bello [1, 2, 3] is pertinent, and includes extensive further references. The operating threshold will also be influenced by spurious receiver responses, which in a well designed receiver may almost always be reduced to negligible proportions.

In 1947, Kotelnikov introduced the concept of an ideal receiving system having a minimum possible value of operating threshold in the presence of Gaussian noise and in the absence of fading; this was later published in 1956 in the U.S.S.R. [4]. The operating threshold defined in this Report reduces to that of Kotelnikov's ideal receiving system in the ideal case of a receiving system having adequate gain, no circuit losses, no fading of the wanted signal and with the receiving system, including its antenna, in an external environment having a specified uniform temperature. To characterize how nearly a given receiving system approaches the ideal system, Kotelnikov introduces an efficiency coefficient which is the ratio of the signal power required for the ideal system to the signal power required for the receiving system under consideration.

The primary reason for using a receiving system with a minimum practicable operating threshold is economic. Thus the required transmitter power is directly proportional to the operating threshold and it is often desirable to use a relatively expensive receiving system with a low value of operating threshold so as to reduce the cost of the transmissions. However, in the case of a broadcasting system involving many thousands of receivers for each transmitter, economic considerations will usually dictate the use of the largest practicable transmitter power. It is pointed out in Report 414 that the simultaneous use of the spectrum by the maximum number of simultaneous interference-free users is dependent only upon the *relative* values of the effective radiated powers of the various transmitters and is essentially independent of their magnitudes, provided these are sufficiently large that noise does not limit the reception at any of the receiving locations.

2. Definitions of wanted available signal powers

Available power is the power that would be delivered to a load if its impedance were conjugately matched to the impedance of the source. We will in this section define the wanted signal power P_a available from the equivalent loss-free receiving antenna and the wanted signal power P'_a available from the actual lossy receiving antenna.

In this Report, all signal and noise powers will be expressed in watts. The convention will be adopted of using lower-case letters to denote power in watts and upper-case letters to denote their equivalents in decibels. Thus:

$$P_a \equiv 10 \log_{10} p_a; \; P'_a \equiv 10 \log_{10} p'_a; \; L_{rc} \equiv 10 \log_{10} l_{rc} \text{ (dB)}$$

$$P'_a = P_a - L_{rc} \text{ (dBW)} \tag{1}$$

For a given radio frequency ν, let $Z_{l\nu}$, Z'_ν and Z_ν represent the impedances of the load, of the lossy antenna in its operating environment and of an equivalent loss-free antenna respectively;

$$Z_{l\nu} = R_{l\nu} + iX_{l\nu} \tag{2}$$

$$Z'_\nu = R'_\nu + iX'_\nu \tag{3}$$

$$Z_\nu = R_\nu + iX_\nu \tag{4}$$

where R and X in the above equations represent the resistance and the reactance respectively. Let $p_{l\nu}$ represent the power delivered to the receiving antenna load and write $p'_{a\nu}$ and $p_{a\nu}$ respectively for the available power at the terminals of the actual receiving antenna and at the terminals of the equivalent loss-free receiving antenna. If v'_ν is the actual open-circuit r.m.s. voltage at the antenna terminals, then:

$$p_{l\nu} = (v'_\nu)^2 R_{l\nu} / |Z'_\nu + Z_{l\nu}|^2 \tag{5}$$

When the load impedance conjugately matches the antenna impedance, so that $Z_{lv} = Z'_v$ or $R_{lv} = R'_v$ and $X_{lv} = -X'_v$, the power p_{lv} delivered to the load is a maximum and is then, by definition, equal to the power p'_{av} *available* from the actual antenna:

$$p'_{av} = (v'_v)^2/4R'_v \tag{6}$$

Note that the available power from an antenna depends only upon the characteristics of the *antenna*, its open-circuit voltage v'_v and its resistance R'_v and is independent of the *actual* load impedance. Comparing (5) and (6), we define a mismatch loss factor as:

$$l_{mav} \equiv p'_{av}/p_{lv} = [(R'_v + R_{lv})^2 + (X'_v + X_{lv})^2]/4R'_v R_{lv} \geq 1 \tag{7}$$

such that the power delivered to the load equals p'_{av}/l_{mav}. When the load impedance conjugately matches the antenna impedance, l_{mav} has its minimum value of unity and $p_{lv} = p'_{av}$. For any other load impedance, somewhat less than the available power is delivered to the load.

The power available from the equivalent loss-free antenna is:

$$p_{av} = v_v^2/4R_v \tag{8}$$

where v_v is the open-circuit voltage for the equivalent loss-free antenna.

Comparing (6) and (8), it should be noted that the available power p'_{av} at the terminals of the actual lossy receiving antenna is less than the available power $p_{av} \equiv l_{rcv}p'_{av}$ for a loss-free antenna at the same location as the actual antenna:

$$l_{rcv} = p_{av}/p'_{av} = (R'_v v_v^2)/(R_v v_v'^2) \geq 1 \tag{9}$$

Note that the open-circuit voltage v'_v for the actual lossy antenna will often be the same as the open-circuit voltage v_v for the equivalent loss-free antenna, but each receiving antenna circuit must be considered individually [19].

For amplitude-modulation systems it will usually be convenient to use the carrier power at the discrete frequency ν as a measure of the wanted signal power; in that case (6), (8) and (9), provide adequate definitions. Similarly, in a frequency-modulation system, the wanted signal power will be concentrated at a discrete frequency ν when there is no modulation and this unmodulated carrier power may then be used as a measure of the wanted signal power. In other cases it will often be useful to consider that the wanted signal power is distributed over the frequency band ν_l to ν_m. In this case:

$$p_a = \int_{\nu_l}^{\nu_m} (\mathrm{d}p_{av}/\mathrm{d}\nu)\mathrm{d}\nu = p'_a l_{rc} \tag{10}$$

$$p'_a = \int_{\nu_l}^{\nu_m} (\mathrm{d}p'_{av}/\mathrm{d}\nu)\mathrm{d}\nu = p_a/l_{rc} \tag{11}$$

where the derivatives $(\mathrm{d}p_{av}/\mathrm{d}\nu)$ and $(\mathrm{d}p'_{av}/\mathrm{d}\nu)$ are used to denote the wanted signal power density in watts per 1 Hz. The limits ν_l and ν_m of the integrals (10) and (11) are chosen to include essentially all sidebands of the wanted signal modulation, but ν_l is chosen to be sufficiently large and ν_m sufficiently small to exclude any appreciable harmonic or other unwanted radiation emanating from the wanted signal transmitting antenna.

3. The direct measurement of the operating threshold of a radio-receiving system

For the direct measurement of the operating threshold of a radio-receiving system it is necessary to have a complete system for the transmission of signals typical of those expected to be transmitted in the service under consideration over the actual transmission path, using the actual transmitting and receiving antennae in their actual environments and having a transmitter the output power of which can be adjusted over a wide range of values. With the transmitter power fixed at some convenient initial level, continuous measurements are made of the instantaneous value of the wanted signal power p'_i available from the receiving antenna over a period of time T_i, say one hour or less, which is sufficiently long that the received signal may be expected to fade over ranges typical of the phase-interference fading expected over the propagation path and yet sufficiently short as to eliminate most of the longer term power fading. If p_m' denotes the median value of the instantaneous wanted signal power p_i' available from the actual antenna, $p_m = l_{rc} p_m'$ will then represent the phase-interference median wanted signal power available at the terminals of the equivalent loss-free receiving antenna. Methods for determining l_{rc} are given in Report 112. During this same period of time a measurement is made of the grade of service g expressed in units which are suitable to the particular kind of service under consideration. For a television broadcasting service it has been found convenient [5, 6] to use the six-point scale $g = 0{\cdot}5$ to $1{\cdot}5$ for unusable, $g = 1{\cdot}5$ to $2{\cdot}5$ for inferior, $g = 2{\cdot}5$ to $3{\cdot}5$ for marginal, $g = 3{\cdot}5$ to $4{\cdot}5$ for passable, $g = 4{\cdot}5$ to $5{\cdot}5$ for fine and $g = 5{\cdot}5$ to $6{\cdot}5$ for excellent, and Weaver [7] has used the transformation $G_w = \log_1 [(6-g)/(g-1)]$ to obtain an approximately linear relation between G_w and $P_{mr}(g)$. It appears that there would be advantages in using the transformation $G = \log_{10} [(7-g)g]$ rather than G_m since $G_w = \infty$ for $g = 1$ and $G_w = -\infty$ for $g = 6$. The above procedure is then repeated many times with the transmitter power adjusted by 3 dB increments above and below the initial adjustment until an adequate range of different grades of services g and corresponding values of p_m has been measured. If such direct measurements are carried out over a sufficiently long period of time $T = mT_i$ so as to take into account the entire range of expected values of phase-interference fading, external noise-levels and noise characteristics, then there will be a large number m of values of g corresponding to the various values of p_m and a statistical approach will be necessary to determine a unique relation between p_m and g. The appropriate statistical method to use will differ with the kind of service under consideration. Where the grade of service may be specified simply as the expected, or mean value $\bar{g}$ of g, or $\bar{G}$ of G, it will be satisfactory to obtain a regression relation between the random variable g (or G) and the given values of p_m (or P_m) depending upon which variables yield a more nearly linear relation. This regression relation will determine, for given values of p_m, the expected value $\bar{g}$, i.e. the operating thresholds $p_{mr}(\bar{g})$ are the values of the phase-interference median signal power p_m available from the equivalent loss-free receiving antenna for which the mean grade of service has its required values g. The operating threshold is often expressed in terms of its decibel equivalent $P_{mr}(g)$ in dBW.

The above-described direct method of measurement of $p_{mr}(g)$ may be used for either gain-limited or noise-limited receiving systems but will often be impracticable because of the necessity of making a long series of measurements for each complete operational system or because of the presence of unwanted signals, other than noise, in the passband of the receiving system. The remainder of this document will be concerned with indirect methods for determining the operating threshold and these indirect methods will also be useful in the design of receiving systems.

4. The effective noise bandwidth and operating noise factor of a receiving system

Essentially all of the basic concepts used in this discussion originated in the early papers by Burgess [8], North [9] and Friis [10] and in the discussion by North [11] of the paper by Friis.

An operating noise factor was originally defined in a paper by North [9], and characterizes the performance of the entire receiving system as contrasted to the receiver noise factor which characterizes only the performance of the receiver itself. Later, Norton [12, 13 and 14] and Barsis *et al* [15] gave a more detailed discussion of this factor and designated it as an effective noise figure. This generalised operating noise factor makes appropriate allowance for the external noise picked up by the receiving antenna as well as the noise introduced by the receiver itself, together with the effects of any losses in the antenna circuit and in the transmission line. The purpose of this section is to give this more general formulation for an operating noise factor, f_{op}, of a receiving system, to describe its general properties, and to show how the operating noise temperature, T_{op}, of the receiving system may be determined from f_{op}. In contrast to the approach to unity of the noise factor, f_n, of an essentially noise-free two-port, the operating noise factor, f_{op}, of an essentially noise-free receiving system approaches zero.

The operating noise factor may be usefully defined only for the approximately linear portion of a receiving system and then only if it has adequate gain so that this portion of the receiving system has a well defined effective noise bandwidth. The operating gain, $g_{0\nu}$, of a receiving system at an input CW frequency ν is defined to be the ratio of the *total* signal power, $p_{d\nu}$, *available* at frequencies ν_i at the output of the linear portion of the receiving system to the input CW power, $p_{av} \equiv l_{rc\nu}p'_{av}$, *available* at the terminals of the equivalent loss-free receiving antenna:

$$g_{0\nu} = p_{d\nu}/p_{av} = p_{d\nu}/l_{rc\nu}p'_{av} \tag{12}$$

For a single-converter, superheterodyne, amplitude-modulation receiver, the load of the linear portion of the receiving system is the second detector and $p_{d\nu}$ denotes the *total* signal power *available* to this second detector at the CW output frequencies $\nu_i = w|n\nu \pm m\nu_{os}|$ where w, n and m represent positive integers and ν_{os} is the local oscillator frequency. Thus, in a typical superheterodyne receiver, although most of the output power will appear at the particular output CW frequency $\nu_i = |\nu - \nu_{os}|$ when the input frequency lies within the principal response band ν_a to ν_b of the receiver, a usually negligible additional power will appear in the output at the other values of ν_i by virtue of beats between the m^{th} harmonic of the input CW frequency ν to produce the w^{th} sub-harmonic of an output frequency ν_i. In some rather unusual superheterodyne systems, these latter components of output power can be appreciable in comparison with that at the intermediate frequency $|\nu - \nu_{os}|$. For a tuned radio-frequency receiver, the output frequencies are given by $\nu_i = n\nu$; for a multiple-converter, superheterodyne receiver the output frequencies ν_i are related to the input frequencies ν in a more complicated way. For frequency-modulation receivers $p_{d\nu}$ would be the signal power available to the first limiter.

Note that the power delivered to the actual load will usually be smaller than the power $p_{d\nu}$ available to this load by virtue of a mismatch loss, but the noise power will suffer very nearly the same mismatch loss so that the available signal-to-noise power ratio will not differ appreciably from the delivered signal-to-noise power ratio. It may be noted that the definition recently adopted by the IEEE [16, 17] is in terms of the delivered rather than the available signal-to-noise ratio at the output. The IEEE definition was presumably adopted to eliminate the difficulty involved with the use of available power when the output has a negative resistance and also to allow for noise power reflected from the load. Unfortunately the impedance of the load of the linear portion of the receiver will very often depend strongly on the level of the signal applied to it and thus it seems better to use the available rather than the delivered signal-to-noise ratio in the basic definition and, in those few cases where this is impracticable, to simply specify the impedance of the device used for measuring the predetection output signal-to-noise ratio. This is the procedure which will be adopted in this document.

The concept of operating noise factor will usefully characterize only receiving systems which are approximately linear, i.e. systems for which $p_{d\nu}$ is approximately proportional to p_{av}, so that $g_{0\nu}$ is very nearly constant over a sufficiently large range of values of p_{av} near to the operating threshold $p_{mr}(g)$. The components of output power associated with frequencies ν_i corresponding to w or n different from unity will not be a linear function of p_{av}, but these components will represent a negligible part of the total output power for conventional receiving systems.

Since a receiving system with an appreciable operating gain will have some kind of bandpass characteristic, we will follow North [11] and define its effective noise bandwidth in Hz as:

$$b = \frac{1}{g_0}\int_{\nu_a}^{\nu_b} g_{0\nu}\,d\nu = \frac{1}{hg_0}\int_0^{\infty} g_{0\nu}\,d\nu \tag{13}$$

$$h = \int_0^{\infty} g_{0\nu}\,d\nu \Big/ \int_{\nu_a}^{\nu_b} g_{0\nu}\,d\nu \tag{14}$$

In the above, ν_a and ν_b are chosen to include only the principal response of the receiving system, i.e. a band of frequencies:

- within which $g_{0\nu}$ has its maximum value g_0;
- which is sufficiently wide to that $g_{0\nu a}$ and $g_{0\nu b}$ are negligibly small relative to the maximum value g_0;
- which is sufficiently narrow so that $g_{0\nu}$ exceeds $g_{0\nu a}$ and $g_{0\nu b}$ for all frequencies ν between ν_a and ν_b.

The factor h in (13) and (14) is a measure of any spurious responses which may be present in the receiver. Spurious response power is, by definition, that power in the output associated with output frequencies ν_{is} corresponding to input frequencies ν lying *outside* the principal response band ν_a to ν_b. Note that $h \geq 1$ and may sometimes be greater than 2 for superheterodyne receivers with little or no selectivity at the input to the frequency converter. In such receivers, spurious responses will be generated by cross-modulation of signals or noise at various frequencies appearing on the frequency converter of the receiver. The frequency converter mixes the noise power lying within a differential input frequency band $d\nu = (\nu_1 - \nu_2)$, which lies somewhere within the principal response band ν_a to ν_b, with the oscillator frequency ν_{os} to produce noise in the output in frequency bands $d\nu_i = |\nu_{i1} - \nu_{i2}|$ determined by the relations:

$$\left.\begin{aligned} \nu_{i1} &= w|n\nu_1 \pm m\nu_{os}| \\ \nu_{i2} &= w|n\nu_2 \pm m\nu_{os}| \end{aligned}\right\} \tag{15}$$

Thus, for each input frequency band d_ν lying within the principal response band ν_a to ν_b, there will be a series of output noise responses in the frequency bands $d_{\nu i}$ determined by (15). These noise outputs arise by virtue of beats between the m^{th} harmonic of the oscillator frequency and the n^{th} harmonic of the noise within the band $d\nu$ to produce the w^{th} sub-harmonic of the intermediate frequency. Similarly, the frequency converter mixes the noise power from differential *spurious* input frequency band $d\nu_s = \nu_{s1} - \nu_{s2}$ which lie outside the band ν_a to ν_b to produce noise in the output frequency bands $d\nu_{is} = |\nu_{is1} - \nu_{is2}|$ determined by:

$$\left.\begin{aligned} \nu_{is1} &= w|n\nu_{s1} \pm m\nu_{os}| \\ \nu_{is2} &= w|n\nu_{s2} \pm m\nu_{os}| \end{aligned}\right\} \tag{16}$$

The frequency bands $d\nu_{is}$ will, in general, overlap the frequency bands $d\nu_i$. However the resulting noise power in the receiver output is simply the sum of the output noise powers arising from the several responses. This will usually be true even in the case of atmospheric or impulsive man-made noise prior to detection. After detection the principal and spurious response noise voltages may be correlated and, in fact, this principle is used in the design of impulsive noise cancellation circuitry.

Note that it would have been possible to define the noise bandwidth by integrating over all of the response bands, thus obtaining the larger bandwidth, hb. However, the frequency band in which the noise finally appears, has the bandwidth b, and this is the motivation for the definition (13). Receivers having spurious responses will have noise factors which are larger than the noise factors of corresponding receivers not having such responses when b is defined by (13). If little or no selectivity is available at the frequency converter input, by far the most important spurious response in a single converter superheterodyne receiver lies within the input frequency band $(2\nu_{os} - \nu_b)$ to $(2\nu_{os} - \nu_a)$ but the magnitude of this spurious response may usually be reduced by increasing the selectivity at this point. Most well-designed receivers have values of h less than 1·01 so that the increase in noise factor caused by spurious responses is usually of negligible importance. Although spurious responses for such receivers do not usually increase the receiver noise factor appreciably, they can lead to very poor system performance when strong unwanted signals lie within the spurious response bands.

The operating noise factor, f_{op}, of a receiving system is defined to be the ratio of the CW signal-to-reference noise power ratio $[p_{ao}/(kT_0b)]$ available at the terminals of the equivalent loss-free receiving antenna to the corresponding signal-to-noise power ratio $[p_{do}/n_d]$ available to the receiving system load with the CW signal tuned to the maximum response of the receiving system bandpass characteristic and with the receiving antenna in its operating environment at an effective temperature T_a:

$$f_{op} = [p_{ao}/(kT_0b)]/[p_{do}/n_d] \qquad (f_{op} \geq 0) \tag{17}$$

The *reference* noise power, kT_0b, is simply the Johnson noise power available in a band, b, from a resistance at the reference absolute temperature T_0 [18]. In the above, p_{ao} and p_{do} denote the values of p_{av} and p_{dv}, respectively, with the CW signal tuned to the maximum response of the receiving system and k is Boltzmann's constant. When the maximum operating gain, g_0, is substituted for the ratio p_{do} to p_{ao} in (17), the following alternative definition is obtained for the operating noise factor:

$$f_{op} = n_d/g_0 kT_0 b = n_d/kT_0 \int_{\nu_a}^{\nu_b} g_{0\nu} d\nu \tag{18}$$

Note that components of the noise power n_d, available to the load may originate external to the antenna in its radiation resistance at an effective temperature T_a, in the loss resistance component of the antenna circuit at an ambient temperature T_c, in the transmission line at an ambient temperature T_t and in the amplifying circuits of the receiver itself which may be characterized by an effective input noise temperature T_e. Furthermore these noise components may be available to the load not only through the principal but also through the spurious response bands of the receiving system.

The operating noise temperature, T_{op}, of the receiving system is, by definition, simply related to the operating noise factor by:

$$T_{op} \equiv f_{op} T_0 \tag{19}$$

5. The measurement of the effective noise bandwidth and of the operating noise factor

The effective noise bandwidth, b, of the receiving system is measured as follows. Replace the receiving antenna, with an output impedance of $Z'_\nu = (R'_\nu + iX'_\nu)$ by a standard signal generator with an output impedance of $Z_{g\nu} = (R_{g\nu} + iX_{g\nu})$. The signal generator is assumed to have an appropriate attenuator and frequency calibration so that its available CW signal power $p_{g\nu}$ and frequency ν are accurately known over a sufficiently large range of different CW power levels within the frequency range from ν_a to ν_b. Let $Z_{l\nu} = (R_{l\nu} + iX_{l\nu})$ denote the input impedance of the receiving system at the terminals of the receiving antenna, let $l_{ma\nu} \equiv |Z'_\nu + Z_{l\nu}|^2/4R'_\nu R_{l\nu}$ denote the mismatch factor at the frequency ν between the receiving antenna and the receiving system input and let $l_{mg\nu} = |Z_{g\nu} + Z_{l\nu}|^2/4R_{g\nu} R_{l\nu}$ denote the mismatch factor between the signal generator and the receiving system input. Now the CW signal power $p'_{a\nu}$ available from the actual receiving antenna corresponding to a given power $p_{d\nu}$ available to the receiving system load is related in the following way to the signal generator power $p_{g\nu}$ required to make available the same power $p_{d\nu}$ to the load:

$$p'_{a\nu} = p_{g\nu} l_{ma\nu}/l_{mg\nu} \tag{20}$$

The available noise power n_{dg} with the signal generator turned off, and the total available signal plus noise power $(p_{d\nu} + n_{dg})$ with the signal generator turned on, may be measured by replacing the second detector by an intermediate-frequency power meter, such as a bolometer or thermocouple, with its impedance conjugately matched to the receiver output impedance over the output band of the receiver. Loss in measurement accuracy arising from a failure to have the power meter exactly conjugately matched to the receiver output impedance will usually not be very large by virtue of the fact referred to earlier that mismatch losses will be roughly the same for both the signal and the noise. Now let $(p_{do} + n_{dg})$ denote the total signal plus noise power delivered to this matched load with the signal generator turned on and tuned to the maximum response of the receiving system. The attenuator on the signal generator should be adjusted so that its available power p_{g0} is sufficiently large that the corresponding delivered power $(p_{do} + n_{dg})$ is several times as large as n_{dg} but not so large as to overload the receiving system. Let $p_{g\nu}$ denote the value of the available power from the signal generator which is required to keep the output power $(p_{d\nu} + n_{dg})$ constant and equal to the maximum response value $(p_{d0} + n_{dg})$ as the signal generator is tuned across the band ν_a to ν_b; note that $p_{g\nu} \geq p_{g0}$. Now the effective noise bandwidth of the receiving system may be determined from:

$$b = \int_{\nu_a}^{\nu_b} \frac{p_{g0} l_{mao} l_{mg\nu} l_{rco}}{p_{g\nu} l_{ma\nu} l_{mg0} l_{rc\nu}} \cdot d\nu \qquad (p_{d\nu} = p_{d0}) \tag{21}$$

In the special case where the loss resistance of the receiving antenna may be considered to be in series with its radiation resistance, R_{rv}, $l_{rcv} = R'_v R_{rv}$. A more general discussion of methods for estimating l_{rcv}, including the effects of insulator losses, is given by Crichlow [19]. The loss factor l_{rcv} was successfully determined in one case from $l_{rcv} = p_{rv}/(l_v p_{av})$ by measuring the power p_{rv} radiated from a target transmitter and calculating the transmission loss l_v between the target transmitting antenna and the receiving antenna. There appears to be no way of directly measuring l_{rcv} without calculating some quantity such as the radiation resistance R_{rv} or the transmission loss l_v. At the higher frequencies l_{rcv} will often be negligibly different from unity; however, in the case of reception with a unidirectional rhombic antenna terminated in its characteristic impedance, l_{rcv} may be greater than 2 [20, 21], since nearly half the received power is dissipated in the terminating impedance and some is dissipated in the ground. For an accurate determination of b, the ratio (l_{mav}/l_{mgv}) must also be known over the frequency range ν_a to ν_b. If the signal generator impedance is made equal to that of the receiving antenna over this frequency range then this factor will be equal to unity. Otherwise it will be necessary to measure Z'_v, Z_{gv} and Z_{iv} and calculate the values of this ratio. If the fractional frequency range $(\nu_a \sim \nu_b)/\nu_a$ is sufficiently small, then a satisfactorily accurate estimate of b may sometimes be obtained by setting all of the loss ratios in (21) equal to unity.

Next, the maximum value g_0 of the operating signal gain is measured as the difference between the measured output total signal plus noise power $(p_{d0} + n_{dg})$ and the measured output noise power n_{dg} divided by the corresponding signal power $p_{a0} = p_{g0} l_{rc0} l_{ma0} / l_{mg0}$ available from the equivalent loss-free receiving antenna:

$$g_0 \equiv \frac{p_{d0}}{p_{a0}} = \frac{[(p_{d0} + n_{dg}) - n_{dg}] l_{mg0}}{p_{g0}\, l_{rc0}\, l_{ma0}} \tag{22}$$

Note that p_{g0} and the ratio (l_{ma0}/l_{mg0}) may be measured, but l_{rc0} is a calculated value.

Finally, with the antenna connected to the input of the receiving system and with no signals present in the principal or spurious response bands, let n_d denote the measured noise power available at the output from the receiving system. In the presence of an appreciable external atmospheric noise level the available noise power n_d must be measured with a power meter having a long time constant of the order of five minutes to obtain a stable value. Using these measured values of b, g_0 and n_d, the corresponding value of f_{op} may be determined from (18). Since the output noise level n_d will, in general, vary with the external noise picked up by the receiving antenna, f_{op} will also vary with this external noise level and statistical methods will be required for an adequate description of the operating noise factor.

To provide further useful relationships and a further insight into the nature of the operating noise factor and its various components, it will be derived in a different way, in Annex I, making use of Friis' theorem for adding the spot noise factors of several two-ports connected in tandem.

6. Dispersed signal source measurement of the noise factor and the effective noise bandwidth of a receiver

In this method, a random-noise signal generator is used, which has its available power dispersed uniformly at least over the principal response band ν_c to ν_d of the receiver and is calibrated in terms of p_g, its available power per unit bandwidth expressed in watts per Hz.

The noise generator, having an ambient temperature T_g and with its output impedance adjusted to be the same as that of the network to which the receiver is connected under operating conditions, is first connected to the receiver. Next the output of the linear portion of the receiver is replaced by a radio-frequency power meter, such as a bolometer or thermocouple, with its impedance conjugately matched to the output impedance of the receiver over the output bandwidth of the receiver. The available noise power n_{dg} with the signal generator turned off and the total available noise power $(p_{dg} + n_{dg})$ with the signal generator turned on are now measured at the output. The level of p_g from the noise generator at the input is adjusted until the available output noise power $(p_{dg} + n_{dg}) = m n_{dg}$, where m is some convenient known level of the output relative to the value with the signal generator turned off; this relative output level is usually set equal to 10.

Next the noise signal generator is replaced by a CW signal generator, also having an ambient temperature T_g and with its output impedance adjusted to be the same as that of the network to which the receiver is connected under operating conditions. With this CW signal generator tuned to the maximum response of the receiver and turned off, the output noise power should have the same value n_{dg} as before. The CW signal generator is now turned on and its level p_{g0} adjusted until the output signal plus noise power $(p_{d0}+n_{dg}) = mn_{dg}$ where m is the same as before.

With the signal and noise powers adjusted to the levels described above, the following relations are obtained:

$$p_{g0}g_r = p_{d0} = p_{dg} = \int_0^\infty p_g g_{r\nu} d\nu = p_g h_r \int_{\nu_c}^{\nu_d} g_{r\nu} d\nu \tag{23}$$

$$b_r \equiv \frac{1}{g_r}\int_{\nu_c}^{\nu_d} g_{r\nu} d\nu \equiv \frac{1}{h_r g_r}\int_0^\infty g_{r\nu} d\nu = p_{g0}/p_g h_r \tag{24}$$

$$h_r = p_{g0}/p_g b_r \tag{24a}$$

$$f_r = [n_{dg}/kT_0 \int_{\nu_c}^{\nu_d} g_{r\nu} d\nu] + [1-(T_g/T_0)]h_r = \frac{n_{dg}p_g h_r}{kT_0 p_{dg}} + [1-(T_g/T_0)]h_r$$

$$= \left\{\frac{p_g}{kT_0(m-1)} + [1-(T_g/T_0)]\right\} h_r \tag{25}$$

The small correction term $[1-(T_g/T_0)]h_r$, in (25) would be zero if T_g were adjusted so that $T_g = T_0$ and may normally be neglected. This method of measurement can be used to determine f_r only when b_r has been measured independently in which case h_r can be determined by (24a) and then f_r determined by (25). An accurate value of h_r cannot be determined from (24a) unless the random noise signal generator has its available power dispersed uniformly over a band which is sufficiently wide to cover all of the spurious response bands.

7. The operating signal-to-noise ratio

The operating signal gain, g_s, of the linear portion of the receiving system may be defined as the ratio of the wanted signal power available to the receiver load, to the wanted signal power available at the terminals of the equivalent loss-free receiving antenna:

$$g_s = \frac{p_d}{p_a} = \frac{\int_{\nu_l}^{\nu_m} l_{rc\nu}(dp'_{a\nu}/d\nu) g_{o\nu} d\nu}{\int_{\nu_l}^{\nu_m} l_{rc\nu}(dp'_{a\nu}/d\nu) d\nu} \equiv \frac{p_d}{l_{rc}p'_a} \tag{26}$$

where $(d'p_{a\nu}/d\nu)$ denotes the wanted signal power spectral density at the terminals of the actual lossy receiving antenna and l_{rc} is defined by (11). In the special case of a wanted CW signal tuned to the receiving system maximum response, $g_s = g_0$. The operating signal gain for modulated or for detuned signals will be smaller than g_0. Thus g_s and l_{rc} will depend upon the degree and nature of the modulation and the tuning of the wanted signal to the receiving system response characteristics.

Now the operating ratio, r, of the available signal-to-available noise at the receiver output will be smaller than the tuned CW signal-to-noise ratio $[p_{d0}/n_d]$ by the factor $[g_s/g_0]$ so that:

$$r \equiv [p_{d0}/n_d][g_s/g_0] = [p_a/(f_{op}kT_0 b)][g_s/g_0] \tag{27}$$

Expressed in decibels, the relation (27) between the wanted signal power, P_a, available at the terminals of the equivalent loss-free receiving antenna and the operating signal-to-noise ratio R available at the output of the linear portion of the receiving system may be written:

$$P_a = F_{op} + B + R + G_0 - G_s - 204 \text{ dBW} \tag{28}$$

Where $F_{op} \equiv 10 \log_{10} f_{op}$, $R \equiv 10 \log_{10} r$, $B \equiv 10 \log_{10} b$,
and $10 \log_{10} kT_0 = -204$, when $T_0 = 288{\cdot}37° \pm 0{\cdot}04°$ K
and $k = (1{\cdot}38054 \pm 0{\cdot}00018) \times 10^{-23}$ [35].

Since T_0 must be somewhat arbitrarily chosen in any case, the above value was assigned so that the constant in (28) is equal to 204 for the currently best estimate of k; this choice of 204 dBW is consistent with Report 322. In engineering practice it is more convenient to use this easily remembered even decibel noise level reference -204 dBW or -174 dBm than to adopt the previously proposed values $T_c = 300°$, 290°, or 1° Kelvin as a reference. Note that the use of $T_0 = 288{\cdot}37°$ results in noise factors less than 0·024 dB larger, and thus negligibly different from, those measured relative to the reference temperature $T_0 = 290°$ adopted by the IEEE [16, 17]; however, the use of a reference temperature $T_0 = 1°$ K would lead to noise factors which would be 24·6 dB larger and equal to $10 \log_{10} T_{op}$.

In noise measurements, another useful physical constant is the ratio of kT_0 to the electronic elementary charge ε, and this has the value $kT_0/\varepsilon = 0{\cdot}024849 \pm 0{\cdot}000003$ V.

The difference $[G_0 - G_s]$ between the tuned CW signal gain and the operating signal gain will usually be negligibly small since the receiver passband will normally be designed to have a width somewhat larger than that ideally required for the reception of the wanted signal. This small difference could be absorbed in the definition of the effective noise bandwidth B but then B would depend upon the wanted signal characteristics as well as the characteristics of the receiving system and this seems undesirable. For this reason this small correction is given explicitly.

A temperature in degrees Kelvin is related to temperatures in degrees Celsius (Centigrade) or in degrees Fahrenheit by:

$$T_{\text{KELVIN}} = 273{\cdot}16 + T_{\text{CELSIUS}} = 255{\cdot}38 + (5/9)T_{\text{FAHRENHEIT}} \tag{29}$$

If the signal generator used for receiver noise factor measurements has its impedance at a temperature T_g rather than T_0, then a term $[1-(T_g/T_0)]h_r$ should be added to the value so measured to determine f_r. It follows that an error in f_r of less than $\pm 0{\cdot}1 h_r$ will be made if T_g lies within the range 260° K to 317° K, i.e., within the range $-13°$ C to 44° C or within the range 8° F to 111° F. Thus the use of the precise value $T_g = 288{\cdot}37°$ K $= 15{\cdot}21°$ C $= 59{\cdot}38°$ F will be required only in connection with very precise noise factor measurements. The IEEE choice of $T_0 = 290°$ [16, 17] was based simply on the desire to have a reference temperature well within the range of temperatures likely to be found in the laboratory where noise factor measurements are made. In this Report the reference temperature $T_0 = 288{\cdot}37°$ K has been chosen since it not only satisfies this requirement but has the further advantages:

- it is based on one of the fundamental constants of physics;
- it leads to the systems equation (28) with the simple constant 204 dBW; and this is consistent with Report 322.

Using the definitions given in Recommendation 341 for the phase-interference median transmission loss L_m, the phase-interference median basic transmission loss L_{bm} and the path antenna gain G_p, the following expressions relate the power P_a available from the equivalent loss-free receiving antenna to the power P_t radiated from the transmitting antenna:

$$P_a = P_t - L_m = P_t + G_p - L_{bm} \tag{30}$$

The radiated power p_t is less than the power input p'_t to the transmitting antenna by a loss factor l_{tc} which makes allowance for the antenna circuit losses.

$$L_{tc} \equiv P'_t - P_t \tag{31}$$

Furthermore the power input p'_t to the transmitting antenna is less than the power p''_t delivered by the transmitter to the transmission line by a loss factor l_{tt} which includes transmission line and mismatch losses:

$$L_{tt} = P''_t - P'_t \tag{32}$$

Combining the above yields the following general formula for the transmitter power required to provide a given operating signal-to-noise ratio R:

$$P''_{tr} = L_{tt} + L_{tc} + L_{bm} - G_p + F_{op} + R + G_0 - G_s + B - 204 \quad \text{dBW} \tag{33}$$

8. The statistical characteristics of the noise and signal envelope voltages at the predetection receiver output

The operating noise factor, operating noise temperature and operating signal-to-noise ratio depend only on the noise and signal powers at the output of the receiving system and these in turn are affected primarily by the effective noise temperature of the antenna, the power gain characteristics of the receiving system and its effective noise bandwidth.

However, the operating threshold of the receiving system also depends upon the following characteristics of phase-interference fading:

- the cumulative amplitude probability distribution (APD) of the noise and signal envelope voltages;
- the expected number of crossings, $n(\Delta)$, per unit of time of various levels Δ by the noise and signal envelope voltages;
- the cumulative distribution of the time durations (DPD) that the noise and signal envelope voltages are above various voltage levels.

If the envelope voltages of the noise and signal could be considered to be randomly distributed in time, the above three statistics would provide a description of their characteristics adequate for most applications. However, atmospheric noise, some kinds of man-made noise, and some kinds of signals tend to occur in bursts of more-or-less continuous noise with varying intervals between these bursts. These characteristics of the noise and signals can also affect the operating threshold of the receiving system, but little progress has so far been made in developing suitable methods of analysis of these particular characteristics although it seems clear that the autocorrelation function of the envelope voltage should provide a useful tool.

The level of the noise or signal envelope voltage may be expressed in decibels above some convenient specified reference level such as its r.m.s. value, its median value, or its mean value over the comparatively short periods of time T_i, say one hour or less, during which the signal or noise envelopes may be considered to be stationary time series [33, 56] and all of the statistics discussed in this section of this Report correspond to this case. The longer term variations of the signal and noise powers are treated by the methods given in Report 414.

In Report 322, Δ represents the level of the envelope voltage above its r.m.s. value expressed in decibels.

Let $q(\Delta)$ denote the probability that $\Delta > \Delta(q)$ and then either $q(\Delta)$ or $\Delta(q)$ can be called the amplitude probability distribution of the envelope voltage for the period of time T_i.

The predictions of the APD for atmospheric noise given in Fig. 27 of Report 322, are expressed in terms of the ratio V_d, expressed in decibels, of the root-mean-square envelope voltage to the average noise envelope voltage, and the effective noise bandwidth, b, of the receiving system in Hz.

Receiving systems with large bandwidths are not adequately characterized by their effective noise bandwidths, b, and, for such systems the impulse noise bandwidth should also be available.

The impulse noise bandwidth of the receiving system is defined by:

$$1/b_i = (1/v_{max}) \int_0^\infty v(t)\mathrm{d}t \qquad \text{(seconds)} \qquad (34)$$

where $v(t)$ is the envelope voltage generated at the predetection output of the receiving system by a single short pulse of voltage introduced into the receiving antenna circuit. If the pulse has a duration of $\Delta t \leqq 1/(10b_i)$, the measured values of b_i will be found to be independent of Δt. Note that the impulse noise bandwidth will be somewhat larger than the effective noise bandwidth for most receiving systems.

The parameter V_d will undoubtedly depend upon:

- the distribution of the time intervals τ_i between the noise impulses in the receiving antenna circuit;
- the distribution of their amplitudes;

– to some extent on the ratio b_i/b;

– more generally, on the shape of the passband.

When $\tau_i \ll 1/b$, V_d will approach its minimum value 1·049 dB and the APD will approach that expected for a Rayleigh distribution [36, 37, 38, 39 and 40].

The median values, V_{dm}, for atmospheric noise for which predictions are given in Report 322 correspond to a receiving system with an effective noise bandwidth $b = 200$ Hz but a method is also given in Fig. 26 of that Report for estimating V_d for atmospheric noise received in systems having effective noise bandwidths $b < b_m$ where b_m is of the order of 10 kHz; Spaulding, Roubique and Crichlow [41] provide a graph for estimating V_d for bandwidths $b < 200$ Hz which is somewhat more convenient to use than Fig. 26 of Report 322. Note that this simple bandwidth transformation will be strictly valid only when the shapes of the response bands are identical. For one of the ARN-2 noise recorders used for determining the noise data reported in Report 322, the ratio $b_i/b = 1{\cdot}225$.

When the resulting calculated values are sufficiently large [$V_d \geq 12$ dB] the following formula may be used to estimate V_d for a receiving system having an effective noise bandwidth within the range $b < b_m$.

$$V_d = V_{dm} - 23 + 10 \log_{10} b \qquad (V_d > 12;\ b < b_m) \qquad (35)$$

The values of V_{dm} to be used in (35) correspond to a receiving system with $b = 200$ Hz and are given as a function of the radio-frequency, time of day, and season of the year in Report 322. With impulse noise bandwidths wider than b_m, it has been found that V_d no longer increases with increasing b in accordance with (35) but begins to level off by virtue of the fact that the peak values of the non-overlapping noise pulses are no longer proportional to b in this case. Let τ_r denote the rise-time of a typical non-overlapping noise pulse in the receiving antenna circuit and the maximum effective noise bandwidth b_m at which V_d is proportional to $10 \log_{10} b$ will, in turn, be proportional to $1/\tau_r$; τ_r and thus b_m will vary with the kind of noise involved and may very well be different for many man-made noise sources than for atmospheric noise.

It has been found, however, that the three parameters V_d, b and b_i are useful for characterizing man-made noise at radio frequencies $\nu < 20$ MHz and bandwidths $b < b_m$ and it is expected that this same approach can be extended to still higher frequencies and to wider bandwidths. The only expected changes in the method of predicting the APD will involve a different relation between V_d and b than that shown in Fig. 26 of Report 322 and a difference in the shape of the APD for $b > b_m$; this difference in shape may also be expected to depend on b_i/b. For radio frequencies $\nu > 2$ GHz man-made noise will usually be negligible and the noise envelope voltage can then be assumed to be Rayleigh distributed.

For the measurement of man-made noise C.I.S.P.R. has somewhat arbitrarily standardized on quasi-peak noise measurements: for the band $\nu = 0{\cdot}15$ MHz to 30 MHz, $b_6 = 9$ kHz, 1 ms charge and 160 ms discharge time constants; and for the band $\nu = 25$ MHz to 300 MHz, $b_6 = 120$ kHz, 1 ms charge and 550 ms discharge time constants. Here b_6 denotes the bandwidth at which the response is 6 dB below the maximum response. However, the motivation in this case is for the development of an objective standard method of noise measurement which can be used for the determination of the degree to which noise suppressors are successful in suppressing the noise from particular noise sources. For this application the C.I.S.P.R. standards are quite appropriate. For the solution of C.C.I.R. frequency assignment problems, however, it is desirable to predict the noise level distribution at particular receiving locations as caused by a representative sample of noise sources and, for such applications, it is desirable to measure the noise power level as discussed in this Report together with the additional statistical parameters described in Report 322 and in this section of the present Report.

Rice [42, 43] has shown that the crossing rate $n(\Delta)$ of the noise envelope voltage can be expressed in the form:

$$n(\Delta) \doteq \alpha b p(\Delta) \qquad \text{Hz} \qquad (36)$$

where $p(\Delta)$ denotes the probability density of the envelope at the level Δ and α is a non-dimensional constant of the order of unity which depends on the shape of the receiver bandpass characteristic and the power spectrum of the noise within this band. Rice also shows how $p(\Delta)$ is influenced by the presence of a sine wave signal within the passband for the particular case in which the APD is Rayleigh distributed.

Let $t(\Delta)$ denote the length of time that the noise envelope voltage exceeds Δ for a particular noise pulse. The mean value of the random variable $t(\Delta)$ is given by:

$$\bar{t}(\Delta) = q(\Delta)/n(\Delta) \text{ (seconds)} \tag{37}$$

The above analysis of the noise envelope voltage has been applied by Norton *et al* [44] to the study of the fading rate of the signal envelope voltage of a fading signal. In this case the effective bandwidth of the propagation medium is usually the controlling factor rather than the bandwidth of the receiving system.

Rice [44, 45] has analysed the DPD, or time duration probability distribution, of the random variable $t(\Delta)$ for noise and signals fading in accordance with the Rayleigh distribution.

The uses for the above statistical descriptions of the wanted signal and of the noise for determining the operating threshold of a radio receiving system will vary with the kind of wanted signal and the kind of noise under consideration and are thus beyond the scope of this Report. Examples of the application of such statistics to particular kinds of service are given in papers by Montgomery [46]; Watt *et al* [47]; Barrow [48]; and Spaulding [49].

Conda [50] discusses the effect of atmospheric noise on the probability of error for an NCFSK system and uses the gamma distribution to describe the fading. This family of fading distributions is mathematically convenient for describing a large range of conditions [51].

Report 415 divides the fading into a short-term stationary component described by the Nakagami-Rice distribution and a longer term power fading distribution; the characteristics of the latter distribution are then best determined empirically rather than by attempting to force a fit of such data to a more or less arbitrary mathematical form such as the gamma distribution. Considering the large number of mechanisms which can be responsible for power fading, it should not be surprising that empirical methods are more suitable for such predictions.

9. The operating threshold $p_{mr}(g)$ of a receiving system

The operating threshold of a receiving system and its overall merit as regards its ability to overcome noise may be measured conveniently by:

$$p_{mr}(g) = r_i(g) f_{op} k T_0 b \equiv r_i(g) k T_{op} b \quad \text{(W)} \tag{38}$$

where $r_i(g)$ denotes the values of the product $r(g_0/g_s)$ required for the system under consideration to provide the specified grade of service g. Note that $r_i(g)$ will depend upon:

- the statistical characteristics of the signal and of the noise discussed in § 8;
- the degree and the nature of the modulation of the wanted signal;
- the degree to which the receiver passband matches and is aligned with the spectral characteristics of the wanted signal.

In particular, any drift of the passband will cause g_s to decrease and then $p_{mr}(g)$ and thus $r_i(g)$ will increase correspondingly. In practice it is not necessary to measure g_s, since $r_i(g)$ is most easily determined from directly measured or calculated values of $p_{mr}(g)$ and b together with a measured value of either f_{op} or T_{op} by using the relations:

$$r_i(g) \equiv p_{mr}(g)/(f_{op} k T_0 b) = p_{mr}(g)/(k T_{op} b) \tag{39}$$

The value of $p_{mr}(g)$ may be measured for non-fading wanted signals by changing the wanted signal power p_a, as defined by (10), until the grade of service actually provided is equal to the grade of service specified. For example, a curve might be plotted of the error rate for a teletype receiving system versus the wanted signal power p_a and then $p_{mr}(g)$ will be equal to that value of p_a corresponding to the value of error rate associated with the specified grade of service.

In many applications, particularly where atmospheric noise is involved, T_a and f_a will be quite variable with time, and in such cases it is useful to consider f_{op} and n_d to be random variables and to describe them in terms of appropriate statistical characteristics. In still other applications, such as space-satellite communications, f_{op} and n_d will be found to vary with the orientation of the receiving antenna since T_a, and thus f_{op} and T_{op} will vary as the antenna is pointed in different directions.

Since both the wanted signal and the noise power may vary from minute to minute in a random and unpredictable fashion, it is convenient to include the effects of these short-term variations of p_a, g_s and n_d in $p_{mr}(g)$ and thus in $r_i(g)$. Thus $p_{mr}(g)$ and $r_i(g)$ should be considered to be measured or calculated median values over a short period of time, say one hour, for which the grade of service provided under typical signal and noise fading conditions is just equal to the grade of service specified. To allow for possible effects of receiver drift on $p_{mr}(g)$, its value measured with the receiver tuned may be increased by the factor g_{st}/g_{sd} where g_{st} and g_{sd} are the effective signal gains with the receiver respectively tuned and then detuned by the amount expected to be exceeded under operating conditions for a tolerable specified percentage of the time.

The figure of merit or operating threshold $p_{mr}(g)$ may be expressed in decibels above one watt as:

$$P_{mr}(g) = R_i(g) + F_m + B - 204 \qquad \text{dBW} \qquad (40)$$

In the above F_m denotes the median value of the operating noise factor F_{op}. Suppose now that the value $R_0(g)$ of operating signal-to-noise ratio required to provide the specified grade of service is determined directly at the predetection output of the receiver. In that case it follows from (28) that $P_{mr}(g)$ is given by:

$$P_{mr}(g) = R_0(g) + G_0 - G_s + F_m + B - 204 \qquad \text{dBW} \qquad (41)$$

The above expression is applicable only to "noise-limited" receiving systems with adequate predetection gain. If the directly determined required value $P_{mr}(g)$ of wanted signal power is greater than the value determined by (41), then the receiving system is said to be "gain-limited". For such systems the value of $R_i(g)$ determined by (40) will be greater than $[R_0(g) + G_0 - G_s]$.

In the special case of an amplitude-modulated signal it is usually more convenient to determine the hourly median value of the wanted signal carrier power $P_{mrc}(g)$ required to provide the specified grade of service. If $R_{rc}(g)$ denotes the median value at the predetection output of the operating carrier-to-noise ratio required to provide the specified grade of service, then:

$$P_{mrc}(g) = R_{rc}(g) + F_m + B - 204 \qquad \text{dBW} \qquad (42)$$

The above is applicable only for a "noise-limited" receiving system tuned to the carrier.

10. Reference point for the operating noise factor

The concept of operating noise temperature may be used for specifying the characteristics of a wide range of devices and systems other than radio receiving systems and the information required for describing such operating noise temperatures in a useful and definitive way is discussed in a recent paper by Engelbrecht [52]. The operating noise factor may be used in general to characterize a *specified* portion of an operating system at any specified input plane or set of input terminals. For the purpose of the C.C.I.R., this reference plane for the operating noise factor of a radio receiving system will always, unless otherwise explicitly stated, be considered to be the input to the terminals of the equivalent loss-free receiving antenna. The receiver input terminals might appear to have some advantage as a reference point since the signal-to-noise ratio can be directly measured at this point. However, it is easy to show by means of an example that the use of this point of reference will not yield an operating noise factor which provides an appropriate measure of the performance of the entire receiving system and this, after all, was the only purpose for defining this factor. For simplicity in the following example, the approximate formula (z)* for f_{op} will be used. If an operating noise factor f_o were defined to be the ratio of the available CW signal-to-

* See Annex I.

reference noise power ratio at the output of the transmission line to the available signal-to-noise ratio at the receiver output, then in the special case for which (z) is applicable:

$$f_0 = f_{op}/l_{rc}l_{rt} = [(f_a - 1)/l_{rc}l_{rt}] + f_r \tag{43}$$

Now consider two systems with $f_{a1} = 3$, $l_{rc1} = 2$, $l_{rt1} = 3$, $f_{r1} = 3$, and $f_{a2} = 5$, $l_{rc2} = 4$, $l_{rt2} = 3$, and $f_{r2} = 3$; for these systems $f_{op1} = 20$ and $f_{op2} = 40$ when referred to the terminals of the equivalent loss-free antenna. The first system is twice as good as the second, since the value of the power $p_{mr}(g)$ required to provide the same signal-to-noise ratio at the output is half as large. The factors f_{o1} and f_{o2} which were referred to the output of the transmission line are both equal to 10/3, though the systems clearly do not have the same performance. It is concluded therefore that the only proper reference point for the operating noise factor and for the corresponding operating noise temperature of a receiving system is at the terminals of the equivalent loss-free receiving antenna.

11. Other noise factor considerations

At very high frequencies, or at very low temperatures, the available noise power from a source at absolute temperature T will be less than kTb by the factor $(h\nu/kT)$ [exp $(h\nu/kt) - 1$] as was shown by Niquist [18]; here h denotes Planck's constant. Since $h\nu/kT = 0{\cdot}0479928\ \nu\ (\mathrm{GHz})/T$ [NBS Technical News Bulletin, October 1963], this correction represents a reduction of less than 0·1 dB in the available noise power when ν (GHz)$/T$ is less than 0·9559, i.e. when $\nu < 275$ GHz at the reference temperature T_0 or when $\nu < 9{\cdot}5$ GHz for a temperature $T = 10°$ K. Balazs [53] has shown that the Johnson noise power available from a conductor also depends on the shape of the conductor at very high frequencies.

There has been discussion in the literature [54, 55] of difficulties with "negative" resistances and their associated "negative" temperatures in some types of amplifier; such considerations are important, however, only in the design and description of the various internal components of the receiver.

It is sometimes possible to reduce the operating noise factor f_{op} of a receiving system and thus to improve its performance by rearranging the order of its component parts. To see how this may be accomplished, we may use the formula of Friis for two-ports in tandem. For two-ports p and q, with noise factors f_p and f_q, the two two-ports in tandem with p preceding q will have a noise factor given by:

$$f_{pq} = f_p + (f_q - 1)/g_p \tag{44}$$

Alternatively, if q precedes p, we have:

$$f_{qp} = f_q + (f_p - 1)/g_q \tag{45}$$

From the above we obtain the condition which must be satisfied for $f_{pq} < f_{qp}$:

$$f_p + (f_q - 1)/g_p < f_q + (f_p - 1)/g_q \qquad (f_{pq} < f_{qp}) \tag{46}$$

We will first consider the conditions under which it is advantageous to use a pre-amplifier p at the antenna terminals preceding the transmission line represented by two-port q. In this case $g_q = 1/l_{rt}$ and $f_q = 1 + (l_{rt} - 1)(T_t/T_0)$ so that we may write for the reduction in f with the pre-amplifier preceding the transmission line:

$$\Delta f \equiv f_{qp} - f_{pq} = (l_{rt} - 1)\{(f_p - 1) + (T_t/T_0)(1 - 1/g_p)\} \tag{47}$$

Since Δf is inherently positive, it follows that f will *always* be decreased by having a pre-amplifier precede the transmission line, and this reduction represents an improvement, expressed in decibels, given by

$$\Delta F = -10 \log_{10} [(f_{pq} + \Delta f)/f_{pq}]$$

Consider next the question of which of two amplifiers in a chain should precede the other. In this case we may subtract 1 from each side of (40) and, provided $g_p > 1$ and $g_q > 1$, we may rewrite this inequality in the form:

$$(f_p - 1)/(1 - 1/g_p) < (f_q - 1)/(1 - 1/g_q) \tag{48}$$

For $f_{pq} < f_{qp}$ provided $g_p > 1$ and $g_q > 1$.

If the above inequality is satisfied, it will be advantageous to have the amplifier p precede the amplifier q, other things being equal.

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ANNEX I

ADDITIONAL DERIVATIONS AND RELATIONSHIPS*

1. The spot noise factor of a passive linear two-port

The *available* gain, $g_{n\nu}$, of a passive linear two-port for a CW frequency ν is defined as the ratio of the available CW signal power, $p_{d\nu}$, at the output terminals of the two-port to the available CW signal power, $p_{g\nu}$, at the output terminals of the signal generator:

$$g_{n\nu} = p_{d\nu}/p_{g\nu} \tag{a}$$

Alternatively, the available loss factor, $l_{n\nu}$, of a linear two-port is given by:

$$l_{n\nu} = p_{g\nu}/p_{d\nu} \tag{b}$$

Note that $g_{n\nu}$ and $l_{n\nu}$ inherently contain a mismatch factor $l_{m\nu}$, determined from the output impedance of the signal generator and the input impedance of the two-port, and thus depend upon the generator impedance as well as upon the characteristics of the two-port itself.

The spot noise factor $f_{n\nu}$, of a linear two-port is here defined, following Friis [5], as the ratio of the available CW signal-to-reference noise power ratio $p_{g\nu}/kT_0 d\nu$ at the signal generator terminals to the available CW signal-to-noise power ratio $p_{d\nu}/d_{n0}$, at its output terminals, with the signal generator output impedance at the reference temperature T_0:

$$\left.\begin{aligned} f_{n\nu} &= [p_{g\nu}/kT_0 d\nu]/[p_{d\nu}/dn_0] \\ f_{n\nu} &= dn_0 l_{n\nu}/kT_0 d\nu \\ &= dn_0/(g_{n\nu}kT_0 d\nu) \end{aligned}\right\} (f_{n\nu} \geq 1) \tag{c}$$

The reference noise power $kT_0 d\nu$ in (c), is simply the Johnson noise power available in an infinitesimal band $d\nu$ from a resistance at the reference absolute temperature T_0. Spot noise factors cannot be measured directly since an infinitesimally narrow filter would be required which just accepts the power in the output frequency band $d\nu$ but, since the spot noise factor of a passive linear two-port is usually not very dependent on frequency, its value can be determined approximately with the aid of a reasonably narrow filter.

Note that the spot noise factor $f_{n\nu}$ depends upon the generator impedance as well as upon the characteristics of the two-port itself, since the gain of the two-port, $g_{n\nu}$, depends upon the mismatch between the generator impedance and the input impedance of the two-port. *Thus one cannot meaningfully describe the noise performance of a two-port in terms of its spot noise factor alone without also specifying the impedance of the generator used in determining it.* The operating noise factor automatically includes the generator impedance, which is the antenna impedance in this case, as an integral part of the receiving system, and thereby provides a complete description of the noise performance of a receiving system.

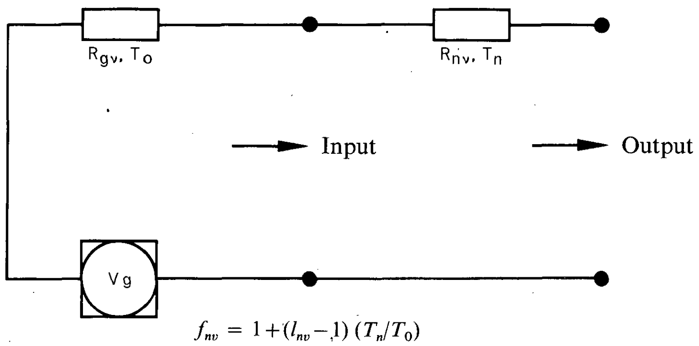
The above definitions will now be used to derive an expression for the spot noise factor of the simple two-port of Fig. 1 with loss $l_{n\nu}$ caused by its resistance, $R_{n\nu}$, at an ambient absolute temperature T_n. Let the resistance, $R_{g\nu}$, of the signal generator be at the reference temperature T_0. The available signal power from the signal generator at the two-port input terminals is given in terms of its open circuit r.m.s. voltage, v_g, by $p_{g\nu} = v_g^2/4R_{g\nu}$ and the available signal power at the output terminals of the two-port is given by $p_{d\nu} = v_g^2/[4(R_{g\nu}+R_{n\nu})]$. Thus, from (b):

$$l_{n\nu} = p_{g\nu}/p_{d\nu} = (R_{g\nu}+R_{n\nu})/R_{g\nu} \tag{d}$$

is obtained.

* Bibliographical references refer to the main Report.

The available noise power at the output of the two-port of Fig. 1 is given by the weighted average of the Johnson noises from the resistances R_{gv} and R_{nv} at temperatures T_0 and T_n:



$$f_{nv} = 1+(l_{nv}-1)\,(T_n/T_0)$$
$$l_{nv} = (R_{nv}+R_{gv})/R_{gv}$$

FIGURE 1

$$\mathrm{d}n_0 = k\mathrm{d}\nu\,\frac{Rg\nu T_0+R_{n\nu}T_n}{Rg\nu+R_{n\nu}} \tag{e}$$
$$= (k\mathrm{d}\nu/l_{n\nu})[T_0+T_n(l_{n\nu}-1)]$$

When this is substituted in (c), the result is:

$$f_{n\nu} = 1+(1_{n\nu}-1)(T_n/T_0) \tag{f}$$

Next, (f) will be derived in another way, and it will be shown that it is applicable in general to any passive two-port with loss l_{nv} and ambient temperature T_n, i.e., one having arbitrary input and output impedances. Note that the available noise output, dn_0, of a linear passive two-port can be expressed as the sum of the two terms:

$$\mathrm{d}n_0 = \mathrm{kd}\nu T_n+k\mathrm{d}\nu(T_0-T_n)/l_{n\nu} \tag{g}$$

The first term would represent the available Johnson noise power from the two-port if its source resistance were also at the ambient temperature T_n, while the second term represents a correction arising from the fact that the temperature T_0 of the source resistance is different from that of the two-port, either higher or lower. For example, suppose that $T_0 > T_n$; in this case the second term in (*g*) is the excess noise power $kd\nu(T_0-T_n)$ available at the input reduced by the factor l_{nv} in passing through the two-port. When dn_0, as given by (g), is substituted in the spot noise factor definition (c), (f) is obtained as before.

Note that the spot noise factor of a passive two-port with its ambient temperature at the reference temperature T_0 is simply equal to its loss factor, i.e., when $T_n = T_0$, it follows from (f) that $f_{nv} = l_{nv}$.

2. The noise factor and effective noise bandwidth of a radio receiver

The noise factor of a radio receiver is measured with a signal generator having a specified output impedance connected to its input; in this document, the output impedance of this generator is taken to be equal to that in the receiving system under consideration.

The receiver gain at a CW frequency ν is defined as the ratio of the total signal power p_{dv} available to the load of the linear portion of the receiving system to the CW power p_{gv} available from the signal generator:

$$g_{r\nu} = p_{d\nu}/p_{g\nu} \tag{h}$$

The effective noise bandwidth, b_r, and spurious response factor, h_r, of the receiver may be defined by:

$$b_r \equiv \frac{1}{g_r}\int_{\nu_c}^{\nu_d} g_{r\nu}\mathrm{d}\nu \equiv \frac{1}{h_r g_r}\int_{0}^{\infty} g_{r\nu}\mathrm{d}\nu \tag{i}$$

where, as for the entire receiving system, ν_c and ν_d are chosen so as to include only the principal response of the receiver and g_r is the maximum value of $g_{r\nu}$.

The noise factor f_r of a radio receiver is defined to be the ratio of the available CW signal-to-reference noise power p_{g0}/kT_0b_r at the terminals of the signal generator to the corresponding total signal-to-noise power ratio p_{d0}/n_d available to the load of the linear portion of the receiver with the CW signal tuned to the maximum response of the receiver bandpass characteristic and with the signal generator output impedance at the reference temperature T_0:

$$f_r \equiv [p_{g0}/kT_0b_r]/[p_{d0}/n_d] \qquad (f_r \geqq h_r) \tag{j}$$

Replacing p_{d0}/p_{g0} in (j) by the maximum gain g_r and b_r by (i), we obtain the following alternative expressions for the receiver noise factor:

$$f_r = n_d/g_r kT_0 b_r = n_d/[kT_0\int_{\nu_c}^{\nu_d} g_{r\nu}\mathrm{d}\nu] \tag{k}$$

Note that components of the noise power n_d available to the load may originate in the output impedance of the signal generator at the reference temperature T_0 and may be available to the load not only through the principal response but also through its spurious response bands; if these were the only sources of noise power available to the load then f_r would be equal to h_r. However, additional noise will be generated in the amplifying circuits of the receiver and some of this noise will also be available to the load; thus f_r will always be greater than h_r.

Since the gain of the receiver depends upon the degree of mismatch between the output impedance of the signal generator and the input impedance of the radio receiver, the specification of the noise factor of a radio receiver can be made unique only by specifying the output impedance of the signal generator used in measuring this noise factor. If the signal generator is at a temperature T_g, rather than T_0 when n_d is measured to determine the noise factor, then a correction term $[1-(T_g/T_0)]h_r$ should be added to the noise factor so measured.

The bandwidth and noise factor of a radio receiver may be measured in much the same way as is described in § 5 of the Report, but in this case without the complication of having to calculate values of $l_{rc\nu}$.

3. The operating noise factor determined from two-ports in tandem

Using the above definitions and conventions, the operating noise factor f_{op} of the linear portion of the receiving system illustrated in Fig. 2 may now be discussed. Let T_c denote the ambient absolute temperature of the antenna circuit resistance exclusive of its radiation resistance, and (f) may be used to determine the spot noise factor of the antenna circuit two-port:

$$f_{c\nu} = 1+(l_{rc\nu}-1)(T_c/T_0) \tag{l}$$

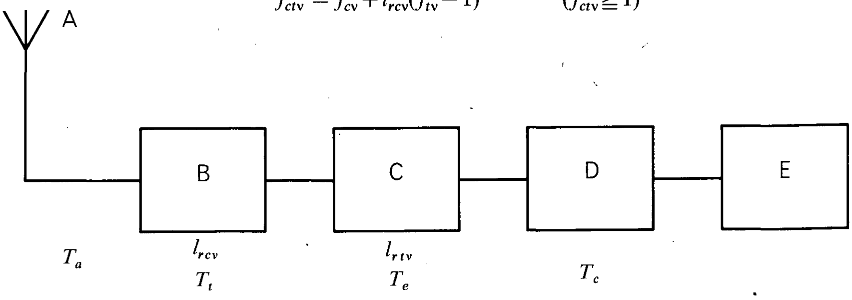
Similarly the spot noise factor of the transmission line two-port with ambient absolute temperature T_t and the available loss factor $l_{rt\nu}$ is given by:

$$f_{t\nu} = 1+(l_{rt\nu}-1)(T_t/T_0) \tag{m}$$

Definitions have been made throughout this document in terms of available signal and noise powers rather than for necessarily matched conditions, since the use of mismatch conditions in the input circuits of the amplifier often leads to a reduction in the operating noise factor of systems containing such amplifiers [22, 23 and 24]. Furthermore it may sometimes be desirable to use non-reflective matching at one end of a long transmission line and the available and delivered powers will, in general, differ somewhat in such cases [25, 26].

Friis [10] gives an expression for the spot noise factor of two two-ports in tandem and this will be used to determine the spot noise factor of the system consisting of the two tandem two-ports (D) and (B) of Fig. 2:

$$f_{ct\nu} = f_{c\nu} + l_{rc\nu}(f_{t\nu} - 1) \qquad (f_{ct\nu} \geqq 1) \tag{n}$$



$$f_a = T_a/T_0; \quad f_{c\nu} = 1 + (l_{rc\nu} - 1)(T_c/T_0); \quad f_{t\nu} = 1 + (l_{rt\nu} - 1)(T_t/T_0); \; f_r$$

$$f_{op} = h(f_a + f_{cta} - 1) + (b_r/b)(g_r/g)(f_r - h_r)$$

$$T_{op} = f_{op}T_0 = h(T_a + T_{ce} + T_{te}) + (b_r/b)(g_r/g)T_e$$

FIGURE 2

The linear portion of a receiving system

A : loss-free antenna with available external noise, $kT_a b$; B : antenna circuit;
C : transmission line; D : receiver;
E : pre-detection output.

It will be convenient to represent the external noise power in the band $d\nu$ which is available at the terminals of the loss-free receiving antenna by $kT_{a\nu}d\nu$ where $T_{a\nu}$ is the noise temperature of the radiation resistance of the receiving antenna at the frequency ν. The concept and method of calculation of an effective temperature $T_{a\nu}$ of the receiving antenna have been described by Slater [27] and by Lawson and Uhlenbeck [28]. Representative values of $T_{a\nu}$ are given by Crichlow [19] and by the C.C.I.R. [29, 30] for frequencies $\nu < 10^8$ and are given by Blake [31] for frequencies within the range $10^8 < \nu < 10^{10}$. Several useful additional sources of information relative to $T_{a\nu}$ are available in the January 1958 Radio Astronomy Issue of the Proceedings of the IRE and in papers by Hansen [32] and Hogg and Mumford [33]. Now the noise power, n_d, delivered to the output of the complete receiving system may be represented, with the antenna at a temperature $T_{a\nu}$ replacing the signal generator having a reference temperature T_0, as the sum of three terms:

$$n_d = k\int_0^\infty T_{a\nu}g_{0\nu}d\nu + kT_0\int_0^\infty (f_{ct\nu} - 1)g_{0\nu}d\nu + kT_0 b_r g_r(f_r - h_r) \tag{p}$$

Since the receiver gain $g_{r\nu}$ is defined as the ratio of the CW power available at its output to the CW power available at its input, then

$$g_{0\nu} = g_{r\nu}/(l_{rc\nu}l_{rt\nu})$$

Now define a weighted average effective antenna noise temperature T_a and a weighted average noise factor f_{cta}:

$$T_a \equiv \frac{\int_0^\infty T_{a\nu}g_{0\nu}d\nu}{\int_0^\infty g_{0\nu}d\nu} \equiv f_a T_0 \tag{q}$$

Note that f_a is the effective antenna noise factor for which predictions are given in Report 322.

$$f_{cta} \equiv \int_0^\infty f_{ct\nu}g_{0\nu}d\nu \Big/ \int_0^\infty g_{0\nu}d\nu \tag{r}$$

Now (p) may be expressed:

$$n_d = kT_0 f_a b h g_0 + kT_0(f_{cta} - 1) b h g_0 + kT_0 b_r g_r (f_r - h_r) \tag{s}$$

When (s) is substituted in the defining equation (18) we obtain the following general formula for the operating noise factor f_{op}:

$$f_{op} = h(f_a + f_{cta} - 1) + (b_r/b)\,(g_r/g_0)\,(f_r - h_r) \tag{t}$$

Let the effective input noise-temperature T_e of the receiver be defined as:

$$T_e \equiv (f_r - h_r) T_0 \tag{u}$$

and let an effective antenna circuit noise-temperature T_{ce} and an effective transmission line noise temperature T_{te} be defined as:

$$T_{ce} \equiv T_c \int_0^\infty (l_{rcv} - 1) g_{0v} dv \Big/ \int_0^\infty g_{0v} dv \tag{v}$$

$$T_{te} \equiv T_t \int_0^\infty l_{rcv}(l_{rtv} - 1) g_{0v} dv \Big/ \int_0^\infty g_{0v} dv \tag{w}$$

Using the above definitions, together with the definition (19), we obtain the following general formula for the operating noise temperature T_{op}:

$$T_{op} = h(T_a + T_{ce} + T_{te}) + (b_r/b)(g_r/g_0) T_e \tag{x}$$

It appears from (x) that the operating noise temperature T_{op} depends not only upon the effective antenna temperature T_a and the effective input noise temperature T_e of the receiver, but also depends upon the losses, mismatch conditions and spurious responses of the receiving system. Thus T_{op} can be identified with an actual temperature only by virtue of the fact that it has the dimensions of a temperature. This follows from the fact that the operating noise factor f_{op} is a dimensionless positive factor which is much greater than unity for typically encountered receiving systems but which may be very much less than unity for microwave receiving systems employing low-noise receiving antennae and masers. In fact, practical receiving systems have been developed with operating noise factors f_{op} substantially less than unity, so that $T_{op} << T_0$ and F_{op} is actually negative; for example, De Grasse *et al* [34] have reported a value of $T_{op} = 18°$ K which corresponds to $F_{op} = -12$ dB.

For most applications it will be satisfactory to assume that $T_c \approx T_0$, $T_t \approx T_0$, $f_{cta} \approx l_{rc0} l_{rt0}$, and $g_r \approx g_0 l_{rc0} l_{rt0}$; with these approximations (t) may be expressed:

$$f_{op} \approx h(f_a - 1) + l_{rc0}\, l_{rt0} [h + (b_r/b)(f_r - h_r)] \tag{y}$$

With the additional approximations $b_r \approx b$ and $h \approx h_r \approx 1$, the operating noise factor may be approximated by:

$$f_{op} \approx f_a - 1 + l_{rc0}\, l_{rt0}\, f_r \tag{z}$$

The above is the expression for the operating noise factor given in Report 322.

Throughout this section, it is understood that all of the spot noise factors such as f_{cv} and f_{tv} and the loss factors such as l_{rcv} and l_{rtv} are determined with generator impedances the same as those in the actual receiving system. Note that Friis' formula for two-ports in tandem is applicable regardless of the degree of match or mismatch between the output impedance of one two-port and the input impedance of a following two-port. The magnitudes of these mismatch losses do, of course, influence the values of the spot noise factors and loss factors and will thus, in turn, affect the operating noise factor.

Equations (t) and (x) may be used to calculate the values of f_{op} and thus of T_{op} in terms of the separately measured values of T_a, T_c, T_t, l_{rcv}, l_{rtv}, h, f_r and h_r; this procedure is especially recommended instead of the direct measurement of f_{op} where f_{op} varies with time or antenna orientation.

ANNEX II

LIST OF SYMBOLS

$p_{mr}(g)$	The operating threshold of a receiving system expressed in watts; the phase-interference median wanted signal power available at the terminals of an equivalent loss-free receiving antenna, which is required to provide a grade of reception g in the presence of noise but in the absence of any other unwanted signals.
$P_{mr}(g)$	The operating threshold of a receiving system expressed in decibels above one watt; $P_{mr}(g) \equiv 10 \log_{10} p_{mr}(g)$.
$p_{mrc}(g)$	The operating threshold of a receiving system expressed in watts of carrier signal power; the phase-interference median wanted carrier power available at the terminals of an equivalent loss-free receiving antenna, which is required to provide a grade of service g in the presence of noise but in the absence of any other unwanted signals.
$P_{mrc}(g)$	The operating threshold of a receiving system expressed in decibels above one watt; $P_{mrc}(g) \equiv 10 \log_{10} p_{mrc}(g)$.
g	The grade of reception.
$f_{op} = (T_{op}/T_0)$	The operating noise factor of a receiving system is the ratio of the CW signal-to-reference noise power ratio $p_{ao}/kT_0 b$ available at the terminals of the equivalent loss-free receiving antenna to the corresponding signal-to-noise power ratio p_{d0}/n_d available to the receiving system load with the CW signal tuned to the maximum response of the receiving system bandpass characteristic and with the receiving antenna in its operating environment.
F_{op} or F_m	Operating noise factor (or median value of the operating noise factor) of a receiving system expressed in decibels.
$f_a \equiv (T_a/T_0)$	Effective receiving antenna noise factor.
$f_{n\nu}$	Spot noise factor of a two-port at the radio frequency ν.
f_r	Noise factor of the receiver with its generator impedance equal to that in the actual receiving system but at a temperature T_0.
$f_{c\nu}$	Spot noise factor of the receiving antenna circuit.
$f_{t\nu}$	Spot noise factor of the transmission line to the receiver with its generator impedance equal to that in the actual receiving system but at a temperature T_0.
$f_{ct\nu}$	Spot noise factor of the receiving antenna circuit and transmission line in tandem.
f_{cta}	Weighted average noise factor of the antenna circuit and transmission line in tandem.
T_0	The value of T_0 for which $10 \log_{10} (kT_0) = -204{\cdot}00$; $T_0 = 288{\cdot}37$ °K $= 15{\cdot}21$ °C $= 59{\cdot}38$ °F.
$T_{a\nu}$	Equivalent loss-free receiving antenna noise temperature in degrees Kelvin at the radio frequency ν.
T_a	Effective antenna noise temperature in degrees Kelvin for a particular receiving system with effective noise bandwidth b and spurious response factor h.

$T_{op} \equiv f_{op} T_0$	Operating noise temperature in degrees Kelvin for a particular receiving system operating in the presence of a particular external noise level.
$T_e \equiv (f_r - h_r) T_0$	Effective input noise temperature of a receiver with noise factor f_r and spurious response factor h_r.
T_c	Ambient temperature in degrees Kelvin of the receiving antenna circuit.
T_{ce}	Effective ambient antenna circuit noise temperature in degrees Kelvin.
T_t	Ambient temperature in degrees Kelvin of the transmission line to the receiver.
T_{te}	Effective ambient transmission line temperature in degrees Kelvin.
T_g	Ambient temperature in degrees Kelvin of the signal generator output impedance.
T_i	A short period of time, say one hour or less, which is sufficiently long so that the received signal may be expected to fade over ranges typical of the phase-interference fading expected over the propagation path and yet sufficiently short so as to eliminate most of the longer term power fading.
T	A period of time sufficiently long to include representative samples of phase-interference fading and external noise levels and characteristics; this period should be representative of the time block for which the service is intended to be available, e.g. the normal working hours from, say, 0800 to 1700 hours, all hours of the year, etc.
p_i	Instantaneous power in watts, a mean value averaged over a period of the radio wave so as to eliminate the pulsations in received power associated with the factor $\cos^2(\omega t)$; variations in p_i occur as a result of both phase-interference and long term variations in the transmission loss.
p_m	Median value of only those variations of the instantaneous power associated with phase-interference; p_m also varies with time and is an approximately log-normally distributed random variable.
$p_{l\nu}$	Power delivered to the receiving antenna load in watts at a radio frequency ν.
p_a	Wanted radio frequency signal power in watts available at the terminals of an equivalent loss-free receiving antenna.
$p_{a\nu}$	Signal power in watts at a radio frequency ν available at the terminals of an equivalent loss-free receiving antenna.
$\mathrm{d}p_{a\nu}/\mathrm{d}\nu$	Wanted radio frequency signal power density in joules (watts per hertz) available at the terminals of an equivalent loss-free receiving antenna.
p'_a	Wanted radio frequency signal power in watts available at the terminals of the actual receiving antenna.
$p'_{a\nu}$	Signal power in watts at a radio frequency ν available at the terminals of the actual receiving antenna.
$\mathrm{d}p'_{a\nu}/\mathrm{d}\nu$	Wanted radio frequency signal power density in joules (watts per hertz) available at the terminals of the actual receiving antenna.
$p_{d\nu}$	Signal power in watts available to the load of the linear portion of the receiving system corresponding to an input frequency ν.
p_{d0}	Signal power in watts available to the load of the linear portion of the receiving system when a CW source of signal power is tuned to the maximum response of the receiving system.

$p_{dg}+n_{dg}$	Noise power in watts available to the load of the linear portion of the receiver when a noise signal generator with noise power density p_g and ambient temperature T_g is connected to its input.
$p_{g\nu}$	Signal power in watts available from a signal generator at the CW frequency ν.
p_{g0}	Signal power in watts available from a CW signal generator tuned to the maximum response of the receiving system.
p_{a0}	Signal power in watts which would be available from an equivalent loss-free receiving antenna when the signal generator has the available power p_{g0}.
p_g	Noise power density in joules (watts per hertz) available from a noise signal generator with its output power dispersed uniformly over a wide range of frequencies.
p_t	Signal power in watts radiated from the transmitting antenna.
p'_t	Signal power in watts delivered to the transmitting antenna.
p''_t	Signal power in watts delivered by the transmitter to the transmission line associated with the transmitting antenna.
p''_{tr}	Transmitter signal power in watts required to provide a given output signal-to-noise ratio r.
r	Ratio of the operating available signal-to-available noise at the output of the linear portion of the receiving system.
$r_0(g)$	The median value of r required for the system under consideration to provide the specified grade of service g; $r_0(g) = r_i(g)\,(g_s/g_0)$.
$r_i(g) \equiv \dfrac{p_{mr}(g)}{f_{op}kT_0b}$	Ratio of the median wanted signal power p_m to operating noise power required at the terminals of the equivalent loss-free receiving antenna to provide the specified grade of service in the absence of all other sources of interference.
$R_{rc}(g)$	The median value of the operating carrier-to-noise ratio at the output of the linear portion of the receiving system which is required to provide the grade of service g; also $R_{rc}(g) \equiv 10\,\log_{10}[p_{mrc}(g)/f_{op}kT_0b]$ and thus has the same value at the input.
n_d	Noise power in watts available to the predetection output of the receiving system.
n_{dg}	Noise power in watts available to the predetection output of the linear portion of the receiving system with the antenna replaced by a standard signal generator with its impedance at a temperature T_g.
$dn_0/d\nu$	Noise power density in joules (watts per hertz) available at the output of a linear two-port.
$g_{0\nu}$	Operating gain of a receiving system; ratio of the total signal power available at the output of the linear portion of the receiving system to the power available at a CW frequency ν at the terminals of the equivalent loss-free receiving antenna.
ν	Radio frequency expressed in hertz.
ν_0	Frequency at which the receiving system has its maximum response, i.e. at which $g_{0\nu}$ has its maximum value.
ν_{os}	The local oscillator frequency expressed in hertz.

g_0	Maximum value of $g_{0\nu}$ within the pass-band of the receiving system, i.e. the value of $g_{0\nu}$ at $\nu = \nu_0$.
g_s	Operating signal gain of a receiving system; the ratio of the total signal power delivered to the load to the wanted signal power available at the terminals of the equivalent loss-free receiving antenna.
$g_{n\nu}, g_{r\nu}$	Available gain of a two-port (or a receiver) at the radio frequency ν; ratio of the total signal power available at the output of a two-port to the power available at its input at a CW frequency ν.
g_n, g_r	Maximum value of $g_{n\nu}$(or $g_{r\nu}$).
$l_{n\nu}$	Available loss factor for a two-port at the radio frequency ν; ratio of the power available at its input at a CW frequency ν to the total signal power available at its output.
$l_{rc\nu}$	Receiving antenna circuit available loss factor at the radio frequency ν.
l_{rc}	Effective available loss factor for the receiving antenna circuit.
$l_{rt\nu}$	Available loss factor for the transmission line to the receiver at the radio frequency ν.
$l_{tc} \equiv p_t'/p_t$	Ratio of the transmitter power delivered to the transmission line to the total power radiated from the transmitting antenna at $\nu_l < \nu < \nu_m$.
v_ν	Open circuit r.m.s. voltage at the terminals of the equivalent loss-free receiving antenna.
v_ν'	Open circuit r.m.s. voltage at the terminals of the receiving antenna in its actual operating environment.
$Z_{g\nu} = R_{g\nu} + iX_{g\nu}$	Signal generator output impedance expressed in ohms.
$Z_\nu = R_\nu + iX_\nu$	Impedance expressed in ohms of the equivalent loss-free receiving antenna.
$Z_\nu' = R_\nu' + iX_\nu'$	Impedance of the lossy receiving antenna in its actual operating environment.
$Z_{l\nu} = R_{l\nu} + iX_{l\nu}$	Input impedance expressed in ohms of the receiving system at the terminals of the receiving antenna.
$R_{r\nu}$	Antenna radiation resistance expressed in ohms at the radio frequency ν.
$l_{ma\nu} = \|Z_\nu' + Z_{l\nu}\|^2/4R_\nu' R_{l\nu}$	Mismatch factor at the frequency ν between the receiving antenna and the receiving system input.
$l_{mg\nu} = \|Z_{g\nu} + Z_{l\nu}\|^2/4R_{g\nu} R_{l\nu}$	Mismatch factor at the frequency ν between the signal generator and the receiving system input.
b, b_r	Effective noise bandwidth of the linear portion of the receiving system (or of the receiver) expressed in hertz.
b_i	Impulse noise bandwidth of the linear portion of the receiving system expressed in hertz.
h, h_r	Spurious response factor of the receiving system; (or of the receiver); also used to denote Planck's constant.
k	Boltzmann's constant.
$f_{op} k T_0 b = k T_{op} b$	Operating noise power available at the terminals of the equivalent loss-free receiving antenna; a fictitious power which cannot be directly measured at this point.

$204{\cdot}00 = -10 \log_{10} (kT_0)$	The Johnson noise power density in decibels below one joule (1 W per hertz) available from a resistance at the reference temperature T_0.
$L_s \equiv 10 \log_{10}(p'_t/p'_a)$	System loss between the actual transmitting and receiving antennae used on the propagation path.
$L_m \equiv 10 \log_{10} (p_t/p_a)$ $= L_b - G_p$	The phase-interference median transmission loss over the propagation path between loss-free antennae which are otherwise equivalent to the actual transmitting and receiving antennae.
L_{bm}	The phase-interference median basic transmission loss; the phase-interference median transmission or system loss expected over the propagation path when the actual antennae are replaced at the same locations by hypothetical antennae which, for all important propagation directions: – are isotropic, so that the directivity gain expressed in decibels is equal to zero in all directions; – have no antenna circuit losses; – have no polarization coupling loss; – have an isotropic phase response.
$G_p \equiv L_{bm} - L_m$	Path antenna gain.
APD	Amplitude probability distribution.
DPD	Time duration probability distribution.
Δ	The level of the envelope voltage of a noise or of a fading signal above its r.m.s. value expressed in decibels.
$q(\Delta)$	The probability that $\Delta > \Delta(q)$.
V_d	The ratio expressed in decibels of the root-mean-square envelope voltage to the average envelope voltage of the noise.
V_{dm}	The median value of V_d for a receiving system with an effective noise bandwidth $b = 200$ Hz; predictions of V_{dm} are given in Report 322.
b_m	The maximum value of b for which the *APD* predictions in Report 322 are valid; $b_m \approx 10$ kHz.
τ_i	Time interval in seconds between successive noise impulses in the receiving antenna circuit.
τ_r	Rise-time of a non-overlapping noise impulse in the receiving antenna circuit.
$p(\Delta)$	The probability density of the envelope at the level Δ.
$n(\Delta)$	The expected number of crossings at the level Δ of the noise or signal envelope voltage.
$t(\Delta)$	The length of time that the envelope voltage exceeds the level Δ for a particular pulse.

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REPORT 414*

EFFICIENT USE OF THE RADIO-FREQUENCY SPECTRUM

(Resolution 1-1)

(1966)

1. Introduction

Starting with the Table of Frequency Allocations which forms part of the Radio Regulations, it should be possible to develop priorities of usage for each portion of the spectrum for the different radio services **. It should then be possible to improve the efficiency of spectrum utilization by successively satisfying, in order of priority, the maximum number of telecommunication requirements within each category of service, under conditions such that the reception of each wanted signal is not subject to harmful interference. Satisfactory service implies a specified grade of service for a specified required percentage of time during assigned hours and in an assigned frequency band. Harmful interference implies a region of interference or a probability of interference which is not negligible. The purpose of this document is to discuss general concepts such as these, which should lead to methods for describing and improving the efficiency of spectrum utilization.

The appropriate model to be used to describe the effects of fading in an application involving either ionospheric or tropospheric propagation will depend upon some *a priori* knowledge of the particular propagation mechanisms believed to be involved. Report 415 provides brief descriptions of important characteristics and differences between several physical models of phase-interference fading. Derivations given in the references provide in most cases only the expected distributions of the instantaneous root-mean-square radio-frequency voltage.

Fading which is sufficiently rapid to produce a significant amount of amplitude, phase, or frequency modulation, dispersion or distortion within the pass-band of the receiving system will affect operating noise thresholds and/or required signal-to-interference protection ratios, and is identified in this Report as "short-term fading", including phase-interference fading, short-term power fluctuations and polarization fading in the case of ionospheric transmission. A more precise definition and some useful mathematical models are given in Report 415.

2. The system design equations

Let r_u denote the ratio between the median value p_m of the instantaneous values of the wanted and the corresponding median value p_{um} of the instantaneous values of the unwanted signal power available from a loss-free antenna which is otherwise equivalent to the actual receiving antenna.

In this Report the convention is adopted of using lower-case letters to denote power in watts, or power ratios, and capital letters will be used to denote their decibel equivalents; thus $R_u \equiv 10 \log_{10} r_u$, expressed in decibels, and $P_m \equiv 10 \log_{10} p_m$ and $P_{um} \equiv 10 \log_{10} p_{um}$, both expressed in decibels above 1 W.

$$R_u \equiv P_m - P_{um} \tag{1}$$

The median values, P_m and P_{um}, of instantaneous power are taken over short periods of time of the order of one hour or less and are more precisely defined as being the median values of only those fluctuations of the instantaneous received power which are associated with "phase-interference fading".

The wanted power, P_m, available from the equivalent loss-free receiving antenna may be determined from the short-term median transmission loss, $L_m \equiv L_{bm} - G_p$, for the wanted signal propagation path and the power, P_t, radiated from the transmitting antenna of the wanted station:

$$P_m = P_t - L_m \equiv P_t + G_p - L_{bm} \tag{2}$$

* This Report was adopted unanimously.

** These priorities will, in general, include considerations other than those of a purely technical nature.

Similarly, the unwanted power, P_{um}, available from the same equivalent loss-free receiving antenna may be determined from the phase-interference median transmission loss $L_{um} \equiv L_{bum} - G_{pu}$ for the unwanted signal propagation path and the power P_{ut} radiated from the transmitting antenna of the unwanted station:

$$P_{um} \equiv P_{ut} - L_{um} = P_{ut} + G_{pu} - L_{bum} \tag{3}$$

Now we obtain:

$$R_u = P_t - P_{ut} + (L_{um} - L_m) = P_t - P_{ut} + (G_p - G_{pu}) + (L_{bum} - L_{bm}) \tag{4}$$

Note that L_m and L_{um} will, in general, vary with time. As a consequence P_m, P_{um} and R_u will also vary with time.

3. System design procedures for efficient use of the radio-frequency spectrum

In this section, more detailed procedures will be given which describe how complete radio systems should be designed to minimize mutual interference between stations. The use of these procedures is expected to lead to a more nearly optimum use of the radio-frequency spectrum, i.e. to the possibility of satisfying a larger number of telecommunication requirements without harmful interference.

Thus the efficient use of the radio-frequency spectrum requires that frequency assignments be so distributed and radio receiving systems be so designed that the reception of the wanted signals is immune to the greatest degree practicable from interference by unwanted radio signals or by noise occupying the same or other radio-frequency channels. The protection ratio $r_{ur}(g)$ which is required for reception with a grade of service, g, of the information carried by a specified wanted signal, in the presence of a specified unwanted signal, but in the absence of any other simultaneously present unwanted signals or appreciable noise is proposed as an element of a figure of merit for a receiving system. The use of receiving systems having the smallest value of $r_{ur}(g)$ for the kinds of unwanted signals likely to be encountered will permit the same portions of the spectrum to be used simultaneously by the maximum number of users. As regards radio services employing frequency-modulation and/or frequency diversity, where a reduction in $r_{ur}(g)$ for an unwanted signal on the same channel may be achieved by occupying larger portions of the spectrum, the use of the spectrum by the maximum number of users will require a careful balance between reductions in $r_{ur}(g)$ on the same channel and on adjacent channels, taking simultaneous account of other system isolation factors such as geographical separation, antenna directivity and cross-polarization.

Report 415 proposes as an additional figure of merit for a receiving system, the phase-interference median value of the wanted signal-power $p_{mr}(g)$ which is required for the reception with a grade of service g of the information carried by a specified wanted signal in the presence of noise, but in the absence of unwanted signals. In the present Report, short-term fading, of which phase-interference fading is a major component, is used for the determination of the required signal power.

Note that the operating threshold $p_{mr}(g)$ is a measure of the required *magnitude* of the short-term median value of the wanted signal power p_m but r_u involves only the *ratio* p_m/p_{um} of the median values of the wanted and unwanted signal powers. For efficient use of the spectrum by the maximum number of simultaneous users, the transmitting and receiving systems of the individual links should be designed and frequency assignments should be arranged with the primary objective of ensuring that the various values of r_u exceed $r_{ur}(g)$ for a sufficiently large percentage of the time during the intended periods of operation. Next the receiving systems should be designed to have the best operating thresholds and then sufficiently high effective radiated powers should be used so that the short-term median value of the wanted signal powers p_m exceed $p_{mr}(g)$ for the specified required percentage of the time during the intended period of reception at each intended receiving location.

If efficient use of all or any part of the radio-frequency spectrum is achieved in the sense of this Report, reception will, in general, at an acceptable percentage of receiving locations, be limited by unwanted signals other than radio noise. The satisfaction of a maximum number of telecommunication requirements, without harmful interference for any significant percentage of time, can be achieved only gradually because of the large investments in radio systems currently in operation. Nevertheless it seems desirable to have a clear statement of procedures which should be employed whenever opportunities arise to move in the direction of this ultimately desirable goal.

This approach to frequency assignment problems will not be applicable in a few cases, such as the bands allocated either on an exclusive or a shared basis to radioastronomy, but these exceptions merely serve to test the general rule [1, 2, 3 and 4] that efficient use of the radio-frequency spectrum for telecommunications can be achieved only when interference from other signals rather than noise provides the ineluctable limit to satisfactory reception.

It is assumed that a given band of radio-frequencies has been allocated to the kind of radio service under consideration and that the nature and technical characteristics of the services occupying the adjacent frequency bands are also known. Furthermore, it is assumed that the geographical locations of each of the transmitting and receiving antennae are specified in advance together with the *relative* values of the radiated powers from each transmitting antenna and the widths of the radio-frequency channels. For a broadcasting service, the specification of the intended receiving locations can be in terms of proposed service areas. With this information given, use may be made of the following procedures to achieve optimum use of this portion of the spectrum by this particular service as well as by other services which are permitted by the Table of Allocations to use this or adjacent portions of the spectrum on a non-interference basis.

3.1 Mutual interference between stations should be minimized by the appropriate assignment of frequencies to all transmitters of a given service and coequal services, sharing the same frequency bands*. In bands 8 and 9, this may be achieved by the practical application of frequency assignment plans [1].

Interference-free reception consistent with efficient use of the radio-frequency spectrum may be achieved by:

- minimizing mutual interference between stations by the application of an appropriate frequency assignment plan, allowing not only for co-channel interference but also for all other types of possible interference;
- designing systems so that $R_u > R_{ur}(g)$ for each unwanted signal and for the specified required percentage of the time during the intended periods of operation at a specified frequency;
- designing systems, or making changes in existing system design, so that the transmission loss L_m and operating threshold $P_{mr}(g)$ are both minimized to the extent that this is not inconsistent with the above consideration;
- using sufficiently high radiated power so that available wanted signal power P_m is greater than $P_{mr}(g)$ for the required percentage of the time during the intended periods of operation.

3.2 The system loss on each of the wanted signal propagation paths should be minimized; this may be accomplished by:

- maximizing the path antenna power gains for each of the wanted signal propagation paths. The path antenna power gains of the wanted signal propagation paths may be maximized by using the maximum practicable transmitting and receiving antenna gains and by minimizing the antenna circuit and polarization coupling losses;
- appropriate antenna siting;
- the use of the highest practicable transmitting and receiving antenna heights in bands 8 and 9.

3.3 The system loss should be maximized on each of the unwanted propagation paths by minimizing the path antenna power gains for each of the unwanted propagation paths. This may involve:

- the use of high-gain transmitting and receiving antennae with optimum side-lobe suppression and front-to-back ratios;

* Two services sharing the same frequency allocations are coequal if both are primary services.

- the use of alternate polarization, if applicable;
- appropriate antenna siting;
- appropriate shielding on some unwanted propagation paths.

3.4 The required protection ratios $r_{ur}(g)$ for various types of interference should be minimized by:
- appropriate radio system design;
- appropriate channel spacing;
- the use of stable transmitting and receiving oscillators;
- the use of linear transmitting and receiving equipment;
- the use of wanted and unwanted signal propagation paths having the minimum practicable phase-interference fading ranges. In bands 6, 8 and 9 minimum phase-interference fading may be achieved by the use of the maximum practicable transmitting and receiving antenna heights;
- the use of space diversity, time- or frequency-diversity and coding.

3.5 Wanted signal propagation paths should be employed having the minimum practicable long-term power fading ranges. In band 7, minimum long-term fading may be expected on circuits operating near the MUF and in bands 8 and 9 minimum fading may be achieved by the use of the maximum practicable transmitting and receiving antenna heights.

The above procedures should be carried out with various choices of transmitting and receiving locations, relative transmitter powers, and channel spacings, until a plan is developed which provides the required service with a minimum total spectrum usage.

After the unwanted signal interference has been suppressed to the maximum practicable extent by the above methods so that, at each receiving location, each of the values of r_u exceeds the corresponding protection ratio $r_{ur}(g)$ for a sufficiently large percentage of the time, then the following additional procedures should be adopted in order to essentially eliminate interference from noise. Note that the minimization of the system loss on each of the wanted signal propagation paths will already have been achieved to the largest practicable extent in connection with procedures 3.2, 3.4 and 3.5 above.

3.6 Receiving systems should be employed which have the lowest practicable values of operating threshold $p_{mr}(g)$, as a general rule. In some cases, effective methods for rejecting interference involve some increase in the operating noise threshold.

3.7 Finally, sufficiently high transmitter powers should be used (at the optimum relative values determined by the above procedures) so that the wanted signal power p_m will exceed the operating threshold $p_{mr}(g)$, for a sufficiently large percentage of the time during the intended period of operation at an acceptable percentage of receiving locations. During favourable conditions of propagation, it may sometimes be advisable to use lower transmission power.

Recently the Joint Technical Advisory Committee of the Institute of Electrical and Electronics Engineers and Electronic Industries Association has prepared a book [2] on radio spectrum utilization. This book provides an interesting review of this subject intended primarily for the frequency administrator, but points out the dominant importance of technical factors in improved use of the spectrum.

4. The two components of fading

Both the wanted signal power p_i and the unwanted signal power p_{ui} available at the terminals of the equivalent loss-free receiving antenna will usually vary from minute to minute in a random or unpredictable fashion. The short-term fading of this instantaneous received power within periods of time T_i, ranging from a few minutes up to one hour or more, is largely associated with random fluctuations in the relative phase between component waves which arrive at the receiving antenna after propagation via a multiplicity of propagation paths having electrical lengths which vary from second to second and from minute to minute over a range of a few wavelengths. However, part of this short-term fading and all of the long-term variations arise from changes in the root-sum-square value of the amplitudes of the component waves, i.e. in short-term changes in the mean power available from the receiving antenna. In the analysis of short-term fading it is convenient to separate the effects of these phase and root-sum-square amplitude changes on the distribution of the instantaneous received power and, for a fixed transmitter power, on the distribution of the instantaneous transmission loss.

In the case of ionospheric propagation, rotation of the polarization vector contributes an important component to the fading of the signal voltage at the antenna terminals with a linearly polarized antenna.

Reports 266-1 and 237-1 give information on the fading phenomena associated with ionospheric and tropospheric propagation, respectively.

The "instantaneous power" P_i will be defined in the usual way to be a mean value, averaged over a single cycle of the radio-frequency, so as to eliminate the variance of power associated with the extremely short-term pulsations associated with the time factor $\cos^2(\omega t)$. It is convenient to divide this intantaneous power $P_i \equiv 10 \log_{10} p_i$ expressed in decibels above one watt into three additive components:

$$P_i = P_m(50) + Y + Y_i = P_m(50) + [P_m - P_m(50)] + [P_i - P_m] \qquad \text{(dBW)} \qquad (5)$$

In the above, Y_i denotes the short-term fading component including phase-interference between multipath propagation components, Y denotes the long-term fading component arising from changes in the short-term median value of the power P_m, while P_m (50) denotes the long-term median value of the power expressed in decibels above one watt. The long-term period T, for which the median value of the power P_m (50) is defined, may be as short as one hour, or as long as several years but should, in general consist only of those particular hours during which a particular wanted station intends to operate on a particular assigned frequency.

In (5), the third component $Y_i = P_i - P_m$, which represents the short-term fading component, is also expressed in decibels, and is a random variable which describes those usually rapid variations of the received power associated with this type of fading.

Frequently, both $p_{mr}(g)$ and p_{um} will be found to vary more or less independently over wide ranges with time. A good approximation to the percentage of time that objectionable interference is present at a particular receiving location may then be obtained [5] by adding the percentage of time that p_m is less than $p_{mr}(g)$ to the percentages of time that p_m is less than each of the values of $r_{ur}(g) \cdot p_{um}$. When this total time of interference is small, say less than 10%, this will represent a satisfactory estimate of the joint influence of several sources of interference which are present simultaneously. Thus, when the fading ranges of the various sources of interference are sufficiently large so that this latter method of analysis is applicable, the various values of $p_{mr}(g)$ and of $r_{ur}(g) \cdot p_{um}$ will have comparable magnitudes for negligible percentages of the time so that one may, in effect, assume that the various sources of interference occur essentially independently in time.

The ratio R_u will vary with time since it is equal to the difference between $P_m(50) + Y$ and $P_{um}(50) + Y_u$

$$R_u \equiv P_m - P_{um} = P_m(50) - P_{um}(50) + Z \qquad (6)$$

$$Z \equiv Y - Y_u \qquad (7)$$

The random variables Y and Y_u tend to be approximately normally distributed with a positive correlation coefficient ρ which will vary considerably with the propagation paths and the particular time block involved; for the usual time block involving all hours of the day for several years, ρ will usually exceed 0·4, even for propagation paths in opposite directions from the receiving point. Z will exceed $Z_a(p)$ for a percentage of time p where the approximate cumulative distribution function of Z is given by:

$$Z_a(p) = \pm\sqrt{Y^2(p) + Y_u^2(100-p) + 2\,\rho\, Y(p)\, Y_u(100-p)} \qquad (8)$$

In the above the positive value is to be used for $p<50\%$ and the negative value for $p>50\%$; $Z_a(50 = 0$. It follows from (6) and (8) that R_u will exceed $R_{ur}(g)$ for at least a percentage of time p provided that:

$$P_m(50) - P_{um}(50) > R_{ur}(g) - Z_a(p) \tag{9}$$

In some cases it may be considered impracticable to determine the function $R_{ur}(g)$ by adding an appropriate short-term fading allowance to $R_{ur0}(g)$; in such cases it may be useful to use the following approximate relation which will ensure that the instantaneous ratio $R_{ui} > R_{ur0}(g)$ for at least a percentage of time p:

$$P_m(50) - P_{um}(50) > R_{ur0}(g) \pm \sqrt{Z_a^2(p) + Z_i^2(0{\cdot}01p, K, K_u)} \tag{10}$$

The parameter Z_i is defined by (27) in Report 415.

In the above the negative sign is to be used for $p<50\%$ and the positive sign for $p>50\%$. Although (10), or its equivalent, has often been used in past allocation studies, this usage is deprecated, since it does not provide a solution which is as well adapted to the actual nature of the problem. The separation of the fading into its two components Y_i and Y makes more appropriate allowance for the fact that communications at particular times of the day or for particular seasons of the year are more difficult than at other times.

5. The joint effect of several sources of interference present simultaneously

The effects of interference from unwanted signals and from noise have so far been considered in this Report as though each affected the fidelity of reception of the wanted signal independently of the other. Let $p_{mr}(g)$ and $r_{ur}(g)\ p_{um}$ denote power levels which the median value of the wanted signal power p_m must exceed to achieve a specified grade of service, when each of these sources of interference is present alone. To the extent that the various sources of interference have a character approximating that of white noise, then when these sources of interference are present simultaneously this same grade of service may be expected from a wanted signal median level $p_m = p_{mr}(g) + \sum r_{ur}(g) p_{um}$.

An approximate method has been developed [4] for determining for a broadcasting service the distribution with time and receiving locations of the ratio $p_m/[p_{mr}(g) + \Sigma r_{ur}(g) p_{um}]$. Although this approach to the problem of adding the effects of interference will probably always provide a good upper bound to the interference, this assumption that the interference power is additive is often not strictly valid. For example, when intelligible cross-talk from another channel is present in the receiver output circuit, the addition of some white noise will actually reduce the nuisance value of this cross-talk.

6. Frequency assignments

Procedures are described in §3 for achieving efficient use of the spectrum without harmful interference as defined in the introduction. It is now generally recognized that the use of computers is essential for optimizing the assignment of frequencies to various individual users within each category of service including the development of efficient frequency assignment schemes [1, 6, 7 and 8]. Typical inputs and outputs for a computer are given in the Annex.

ANNEX

TYPICAL INPUTS AND OUTPUTS FOR A COMPUTER

1. Nominal frequency assignments.
2. Transmitting system locations including the antenna heights.
3. Transmitting system signatures, i.e. the radiated emission spectrum characteristics including any spurious emission spectra.
4. Transmitting and receiving antenna characteristics.
5. Receiving system locations including the antenna heights.
6. Spurious response and emission spectra of the receiving systems.

7. Operating thresholds of the receiving systems in their actual environments, which thus make appropriate allowance for the effects of both man-made and natural noise.

8. Required values of wanted-to-unwanted phase-interference median signal powers for all unwanted signals which could potentially cause harmful interference to the wanted signal; these protection ratios include appropriate allowances for reductions in the effects of fading achieved by the use of diversity reception and coding.

9. Distribution with time of the phase-interference median basic transmission loss for the wanted and all of the unwanted signal propagation paths.

10. Transmission line and antenna circuit losses.

11. Correlation between the phase-interference median transmission losses on the wanted and on each of the unwanted propagation paths.

12. Path antenna gains for the wanted and all of the unwanted propagation paths; these path antenna gains include allowances for antenna orientation, polarization coupling and antenna-to-medium coupling losses.

13. The spurious emission man-made noise sources such as those noted in Report 182.

14. Assigned hours of operation of each wanted and each unwanted emission.

The output of the computer indicates the identity and nature of the cases of harmful interference encountered. Harmful interference is defined as a failure to achieve at least the specified grade of service for the specified required percentage of the time during the assigned hours of operation at the given frequency. When harmful interference is indicated, the inputs are changed so that by an iterative process an assignment plan is finally achieved with cases of harmful interference.

The task of the radio engineer is to develop efficient radio systems and the principal tool for improving efficiency is to adjust the various " trade-offs " to their optimum values. For example, it is usually more economical to use lower effective radiated powers from the transmitting systems by reducing the operating thresholds of the receiving systems. Receiving system thresholds can be reduced in several ways, for instance, by:

- reducing the level of internally generated noise;
- the use of antenna directivity to reduce the effects of external noise;
- the reduction of man-made noise levels by the use of suppressors on noise generators such as ignition systems, relays, power transmission systems, etc.;
- the use of space diversity, time-diversity and coding;
- the use of more spectrum in a wide-band frequency-modulation system or in a frequency-diversity system. This can also reduce the receiving system operating threshold, and can reduce the protection ratios against unwanted signals.

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REPORT 415*

MODELS OF PHASE-INTERFERENCE FADING FOR USE IN CONNECTION WITH STUDIES OF THE EFFICIENT USE OF THE RADIO-FREQUENCY SPECTRUM

(1966)

1. Introduction

This Report discusses certain models of phase-interference fading which are essential to the work of International Working Party III/1, set up under Resolution 1-1.

2. The Rayleigh distribution Y_i

The short-term phase-interference Rayleigh distribution is the one most commonly observed in both ionospheric and tropospheric propagation. Let the random variable v_{ij} denote the instantaneous open circuit root-mean-square amplitude and let the random variable θ_{ij} denote the instantaneous phase of the j^{th} multipath component ($j = 1$ to n) of the radio-frequency voltage at the equivalent loss-free receiving antenna terminals after adding or subtracting a multiple of 2π so that θ_{ij} lies between $-\pi$ and π. The instantaneous power p_{ij} available at these terminals from this j[th] multipath component is a random variable equal to $v_{ij}^2/4R$; see (8) in Report 413. Let the random variable v_i denote the instantaneous r.m.s. amplitude and the random variable θ_i the instantaneous phase of the *vector* sum of the n multipath voltages $v_{ij}(j = 1$ to $n)$; the instantaneous power p_i available at the terminals of the equivalent loss-free receiving antenna terminals is a random variable equal to $v_i^2/4R$. The expected value $\bar{p}_i$ of this instantaneous power can be shown to be equal to the sum of these n multipath components of power provided the following two conditions are satisfied:

– the random variables θ_{ij} are not correlated, i.e. the expected values of $\{\theta_{ij} \cdot \theta_{ik}\} = 0$ for $j \neq k$ ($j = 1$ to n and $k = 1$ to n);

– the random variables θ_{ij} are uniformly distributed between $-\pi$ and π.

$$\bar{p}_i = \sum_{j=1}^{n} p_{ij} \tag{1}$$

Since each of the components p_{ij} of $\bar{p}_i$ is a random variable, it follows that the expected value p_i will also be a random variable, the variations of which are called long-term power fading.

If, in addition to the two conditions above, the following condition is also satisfied:

$$\text{(c)}\quad p_{ij} << \bar{p}_i \qquad (1 \leqq j \leqq n)$$

then the probability q that $p_i > p_i(q)$ is given by:

$$q = \exp\left[-p_i(q)/\bar{p}_i\right] \tag{2}$$

and all values of the phase θ_i between $-\pi$ and π, after adding or subtracting a multiple of 2π so that θ_i lies in this interval, will be equally likely. This uniform distribution of θ_i and the amplitude distribution (2) completely characterize the Rayleigh distribution. As $p_i(q)$ varies from ∞ to 0, q varies from 0 to 1. If we set $q = 0{\cdot}5$ in (2) it follows that the phase-interference median power is given by:

$$p_m \equiv p_i(0{\cdot}5) = \bar{p}_i \log_e 2 \doteq 0{\cdot}69315\bar{p}_i \tag{3}$$

In terms of the median power p_m the Rayleigh distribution (7) becomes:

$$q = \exp\left(-\frac{p_i(q)\log_e 2}{p_m}\right) \tag{4}$$

The distribution of the instantaneous power is characterized by one parameter, e.g. $\bar{p}_i$ in (2) or p_m in (4).

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* This Report was adopted unanimously.

Suppose now that $z_1, z_2, \ldots z_N$, is a sample of N independent observations of p_i from a Rayleigh distribution of p_i. Siddiqui [11] has shown that $\log_e 2$ times the average to the N values of z_i is the best unbiased estimate of the median power p_m:

$$\hat{p}_m = \left[(\log_e 2)/N \right] \sum_{i=1}^{N} z_i \tag{5}$$

and the variance of this estimate is given by:

$$\sigma^2_{pm} = (p_m)^2/N = (\hat{p}_m)^2/N \tag{6}$$

Siddiqui [11] has also shown that the sample median value $z(0 \cdot 5)$ has a variance given by:

$$\sigma^2_{z(0\cdot5)} = (p_m(\log_e 2)^2/N = 2\cdot0814 p_m^2/N = 2\cdot08[z(0\cdot5)]^2/N \tag{7}$$

One might conclude that the estimate of p_m given by (5) and based on the mean value of the sample is a better estimate than the sample median value $z(0 \cdot 5)$ since the variance of the latter is more than twice as large. Note, however, that the accuracy of the estimate $\hat{p}_m$ also depends on the accuracy of measurement of all N values in the sample whereas $z(0 \cdot 5)$ depends only on measurement accuracy near the median value $z(0 \cdot 5)$. In practice the measurement accuracies of both the large and small values of z_i are often much less than the measurements made near $z(0 \cdot 5)$; this is the reason that the sample median $z(0 \cdot 5)$ is usually preferred by experimenters to $\hat{p}_m$ even though the latter has less than half the variance of $z\ (0 \cdot 5)$.

Consider now the normalized variable $y_i(q) \equiv p_i(q)/p_m$. The probability q that $y_i > y_i(q)$ is given by:

$$q = \exp - [y_i(q) \log_e 2] \tag{8}$$

In decibels, (5) may be expressed:

$$Y_i(q) = 0 \cdot 521390 + 10 \log_{10}\{\log_{10} (1/q)\} \tag{9}$$

where $Y_i > Y_i(q)$ with probability q. Note that the normalized distributions (8) and (9) are free of any parameters.

Returning to (2) note that, as q approaches zero, $p_i(q)$ approaches infinity and it is clear that such large values of p_i will not be observed in practice. The instantaneous maximum value $m(p_i)$ will occur in the exceptional case when θ_{ij} has the same value θ_0 for $j = 1$ to n:

$$\bar{p}_i \leqq m(p_i) = [\sum_{j=1}^{n_1} (p_{ij})^{1/2}]^2 \leqq np_i \tag{10}$$

The upper bound np_i in (10) corresponds to the exceptional case in which all p_{ij} have the same value p_0, whereas the lower bound p_i in (10) corresponds to the exceptional case in which one value of $p_{ij} = p_0$ and the remainder are all equal to zero. For n equal components the maximum value $m(p_i) = np_i$ would be expected, for an actual Rayleigh distribution with $n = \infty$, to be exceeded with a probability $\exp(-n)$. Since $\exp(-5) = 0 \cdot 00614$, it follows that the assumption of a Rayleigh distribution is likely to lead to erroneous conclusions at probability levels of the order of $0 \cdot 005$ and smaller unless n is much greater than 5. This is especially so since these n components are not likely to have comparable magnitude.

3. Other fading distributions

The following formula [12, 13 and 14] gives the probability density of the signal envelope voltage v expected under the assumptions (1) that v is the resultant envelope voltage of n components, each with specified constant envelope voltage $v_j (1 \leqq j \leqq n)$, plus a random Rayleigh-distributed component with r.m.s. amplitude $v_{r.m.s.}$ and (2) that the phase of each constant component is random, i.e., that all values of the phases of the constant components between $-\pi$ and π are equally likely:

$$P(v, v_j, v_{r.m.s.})\mathrm{d}v = \mathrm{d}v \cdot v \int_0^\infty \lambda \exp(-v^2_{r.m.s.}/4) J_0(v\lambda) \cdot \prod_{j=1}^{n} J_0(v_j, \lambda)\mathrm{d}\lambda \tag{11}$$

It can be shown that the second and fourth moments of this distribution are given by:

$$\overline{v^2} = v^2_{r.m.s.} + \sum_{j=1}^{n} v_j^2 \tag{12}$$

$$\overline{v^4} = 2(\overline{v^2})^2 - \sum_{j=1}^{n} v_j^4 \tag{13}$$

Since (11), (12) and (13) give the distribution of the envelope voltage v and since the instantaneous power p_i is proportional to v^2, these equations also give the probability density and moments of the distribution of instantaneous power:

$$p(p_i, p_j, p_R)\mathrm{d}p_i = \frac{\mathrm{d}p_i}{p_R}\int_0^\infty t \exp(-t^2/4) J_0\left(t\sqrt{\frac{p_i}{p_R}}\right)\prod_{j=1}^{n} J_0\left(t\sqrt{\frac{p_j}{p_R}}\right)\mathrm{d}t \tag{14}$$

$$\bar{p}_i = p_R + \sum_{j=1}^{n} p_j \tag{15}$$

$$\overline{p_i^2} = 2(\bar{p}_i)^2 - \sum_{j=1}^{n} p_j^2 \tag{16}$$

The variance σ_{pi}^2 may be obtained from (15) and (16):

$$\sigma_{pi}^2 = (\bar{p}_i)^2 - \sum_{j=1}^{n} p_j^2 \tag{17}$$

In the above p_j denotes the power associated with the constant component j and p_R the mean power associated with the Rayleigh-distributed component.

In practice it is not often feasible to specify the $n+1$ parameters $v_j(1 \leqq j \leqq n)$ and v_{rms} or $p_j(j = 1$ to $n)$ and p_R of the above distributions. However, when there is only a single constant component v_1 (11) becomes:

$$p(v, v_1, v_{r.m.s.})\mathrm{d}v = \frac{\mathrm{d}v}{v^2_{r.m.s.}}\, 2\, v \exp\left[-\frac{v_1^2 + v^2}{v^2_{r.m.s.}}\right] \mathrm{I}_0\left(\frac{2 v_1 v}{v^2_{r.m.s.}}\right) \tag{18}$$

The above probability density function was first derived by Nakagami [14] and independently by Rice [12]. This distribution of the sum of a constant component and a Rayleigh-distributed random component has been extensively studied [15, 16 and 17].

The Nakagami-Rice distribution is more easily related to physical models than distributions such as the gamma distribution, which, however, is mathematically convenient for describing the large range of conditions observed in studies of short-term fading. Short-term and long-term fading are both affected by "phase-interference" and "power" fading mechanisms.

The probability density of the distribution of instantaneous power in the case of the Nakagami-Rice distribution may be expressed:

$$p(p_i, p_1, p_R)\mathrm{d}p_i = \mathrm{d}p_i \exp\left[-\frac{p_1 + p_i}{p_R}\right] \mathrm{I}_0\left(\frac{2}{p_R}\sqrt{p_1 p_i}\right) \tag{19}$$

The mean and variance of the Nakagami-Rice distribution of instantaneous power may be obtained as a special case of (10) and (12):

$$\bar{p}_i = p_R + p_1 \tag{20}$$

$$\sigma_{pi}^2 = p_R + 2p_1 p_R \tag{21}$$

When $p_j << p_R$, $(j = 1$ to $n)$, the distribution (14) reduces to the probability density for the Rayleigh power distribution:

$$p(p_i, \bar{p}_i)\mathrm{d}p_i = \mathrm{d}p_i \exp(-p_i/\bar{p}_i) \tag{22}$$

Experience has shown that the Nakagami-Rice distribution is the one most useful for describing phase-interference fading and, even in this case, it is often difficult to assign values to its two parameters p_R and p_1 which will be appropriate to a particular receiving system operating at a particular geographical location and using a particular radio frequency.

It is often advantageous to express p_1, p_i and p_R in decibels and Burns [18] has studied the distribution of $R \equiv 10 \log_{10} (p_i/p_1)$ using $K \equiv 10 \log_{10} (p_R/p_1)$ as a parameter on the assumption that p_i is distributed in accordance with (19). He obtains explicit formulae for R and σ_R^2 as a function of K.

For the important special case of a Rayleigh distribution, $K = \pm\infty$ and the cumulative probability q that y_i will exceed $y_i(q)$ for a given value, p_m may be expressed by (8). Alternatively:

$$q[p_i > p_i(q)|p_m] = \exp\left[-p_i(q) \log_e 2/p_m\right] \qquad (K = +\infty) \tag{23}$$

$$q[y_i > y_i(q)] = [q Y_i > Y_i(q)] = \exp\left[-y_i(q) \log_e 2\right] \qquad (K = +\infty) \tag{24}$$

$$Y_i(q) = 5{\cdot}21390 + 10 \log_{10}[\log_{10}(1/q)] \qquad (K = +\infty) \tag{25}$$

Fig. 1 and Table I show the Nakagami-Rice phase-interference fading distribution $Y_i(q)$ as a function of K for particular values of q. It is evident from Fig. 1 that the distribution of phase-interference fading depends only on K, the ratio in decibels between the root-sum-square value of the amplitudes of the Rayleigh fading component and the amplitude of the steady component of the received signal. The utility of this distribution for describing phase-interference fading in ionospheric propagation is discussed in Report 266-1 and for tropospheric propagation is demonstrated in [19, 20 and 21].

Fig. 2 shows several examples of phase-interference fading in tropospheric propagation.

On within-the-horizon tropospheric paths, including either short point-to-point terrestrial paths or paths from an earth station to a satellite, K will tend to have a large negative value throughout the day for all seasons of the year. As the length of the terrestrial propagation path is increased or the angle of elevation of a satellite is decreased, so that the path has less than first Fresnel zone clearance, the expected values of K will increase until, for some hours of the day, K will be greater than zero and the phase-interference fading for signals propagated over the path at these times will tend to be closely represented by a Rayleigh distribution. On most beyond-the-horizon paths, K will be greater than zero most of the time. However, when the signals arrive at the receiving antenna via ducts, knife-edges or elevated layers, the values of K may decrease to values much less than zero even for trans-horizon propagation paths. For a given beyond-the-horizon path, K will tend to be negatively correlated with the median power level p_m; i.e., large values of K are expected with small values of p_m. For some within-the-horizon paths, K and p_m tend to be positively correlated.

Discussions of fading rate and the distribution of the duration of fading are given in Report 242 for tropospheric propagation and in Report 266-1 for ionospheric propagation.

Only the short-term variations of p_i and p_{ui} associated with phase-interference fading are used in determining $r_{ur}(g)$, the ratio of the median wanted signal power p_m to the median unwanted signal power p_{um} required to provide the specified grade of service g. Separate account is taken of the variations with time of the median power levels P_m and P_{um}. This separation of the total fading into a phase-interference component Y_i and the more slowly varying component Y appears to be desirable for several reasons:

- those variations of that component of the instantaneous received power, Y_i, which are associated with phase-interference alone may be expected to occur completely independently for the wanted and unwanted signals, and this facilitates making a more precise determination of the proper value for $R_{ur}(g)$;

- the random variable Y_i follows the Nakagami-Rice distribution, as illustrated in Fig. 1, while variations with time of the remaining component Y are approximately normally distributed;

- the variations with time of P_m and of P_{um} tend to be correlated for most wanted and unwanted propagation paths, and an accurate allowance for this correlation is facilitated by separating the instantaneous fading into the two additive components Y and Y_i;
- most of the contribution to the variance of P_m with time occurs at low fluctuation frequencies ranging from one cycle per year to about one cycle per hour, whereas most of the contribution to the variance of Y_i occurs at the higher fluctuation frequencies greater than about one cycle per hour.

A recent report [3], prepared for the International Working Party established by Resolution 2, contains methods for predicting the cumulative distribution of P_m and of P_{um} as well as the values of K expected for wanted propagation paths and the values of K_u expected for unwanted propagation paths; these predictions are applicable to tropospheric propagation, i.e., for frequencies above about 40 MHz. However, the Nakagami-Rice distribution is equally applicable to ionospheric propagation; similar procedures for predicting K for such paths are available in the literature [22].

4. The distribution of the ratio of an instantaneous wanted signal power and an instantaneous unwanted signal power when these random variables are each independently distributed in accordance with the Rayleigh distribution

Let R_{ui} denote the instantaneous ratio between the instantaneous wanted signal power $P_m + Y_i$ and the instantaneous unwanted signal power $P_{um} + Y_{ui}$:

$$R_{ui} = P_m + Y_i - P_{um} - Y_{ui} = R_u + Z_i \tag{26}$$

where

$$Z_i \equiv Y_i - Y_{ui} \tag{27}$$

Now assume that Y_i and Y_{ui} are each independently Rayleigh distributed in accordance with (9). The assumption of independence will almost always be valid in practice since it is most unlikely that the multipath components on the wanted propagation path will be correlated in their behaviour with the multipath components on the unwanted propagation path. The distribution of the random variable Z_i is free of parameters when Y_i and Y_{ui} are Rayleigh distributed and, using results obtained by Siddiqui [23], it is easy to show that the probability q that $Z_i > Z_i(q)$ may be determined in this case from:

$$Z_i(q) = 10 \log_{10} \left(\frac{1}{q} - 1 \right) \tag{28}$$

The expected instantaneous distribution of R_{ui} may be determined simply by adding to $Z_i(q)$ the constant $R_u = P_m - P_{um}$. The mean value $\bar{Z}_i = 0$ and the standard deviation $\sigma_{zi} = 7{\cdot}877$ dB. Siddiqui [23] has shown that the expected value of $z_i \equiv y_i/y_{ui}$ is infinite and this is one of the reasons for using Z_i.

5. Experimental determination of the cumultative distribution of Z_i

Consider the continuous time processes:

$$P_i(t) = Y_i(t) + Y(t) + P_m(50) \tag{29}$$

$$\begin{aligned} Z_e[t, \tau(t)] &= P_i(t) - P_i[t+\tau(t)] \\ &= Y_i(t) - Y_i[t+\tau(t)] + Y(t) - Y[t+\tau(t)] \\ &\approx Y_i(t) - Y_i[t+\tau(t)] \end{aligned} \tag{30}$$

The approximation in (30) may be made since $Y(t)$ varies with time much more slowly than $Y_i(t)$ and this approximation will be better the smaller the value of $\tau(t)$. However, if $\tau(t)$ is made too small, $Y_i(t)$ will be correlated with $Y_i[t+\tau(t)]$ and $Z_e[t, \tau(t)]$ will then not represent a good approximation to $Z_i(t)$ which is, by definition, the difference between two *independent* distributions of $Y_i(t)$.

In order to ensure the independence of $Y_i(t)$ and $Y_i[t+\tau(t)]$ we may determine the autocorrelation of the continuous function $P_i(t)$. Let $\tau(t)$ denote the particular time interval for which $P_i(t)$ and $P_i[t+\tau(t)]$ have an autocorrelation of 0·4 over the time interval from t to $t+10\ \tau(t)$; Siddiqui has shown that $P_i(t)$ and $P_i[t+\tau(t)]$ will then be approximately independent random variables. Note that $\tau(t)$ will vary with the time t; using this variable, as determined above, the continuous process $Z_e[t,\ \tau(t)]$ may be generated and its observed cumulative distribution may be compared with the cumulative distribution expected for some model of the phase-interference fading. For example, it should be compared with $Z_i(q)$ as given by (27) when the Rayleigh distribution is the assumed model. Note that $Z_e[t,\ \tau(t)]$ is almost completely independent of the long-term power fading component Y, since it is in error only by the change in Y over the time interval $\tau(t)$ which, in some cases, may be as small as one second. For this reason $Z_e[t,\ \tau(t)]$ will represent a stationary time series over the period of time during which the model being tested is believed to be applicable. This latter time will often be as large as 3 hours, and, in this case, the number of independent samples of $Z_e[t,\ \tau(t)]$ with $\tau(t) = 1s$ would be 10 800.

TABLE I*(a)*

Characteristics of the Nakagami-Rice phase-interference fading distribution $Y_i(q)$

($Y_i > Y_i(q)$ with probability q; $Y_i(0{\cdot}5) \equiv 0$)

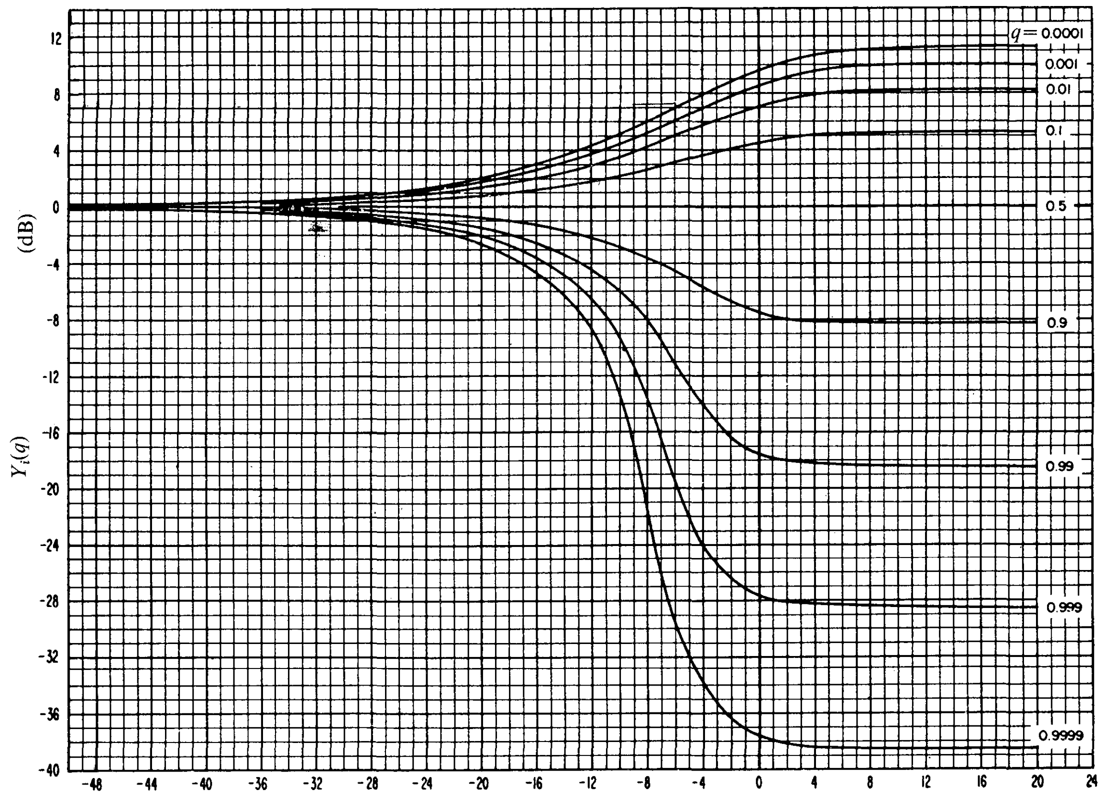
K	$\bar{Y}_i$	σ_{yi}	$Y_i(0{\cdot}01)$	$Y_i(0{\cdot}1)$	$Y_i(0{\cdot}9)$	$Y_i(0{\cdot}99)$	$Y_i(0{\cdot}1) - Y_i(0{\cdot}9)$
dB	dB	dB	dB	dB	dB	dB	dB
−40	−0·0002	0·061	0·1417	0·0784	−0·0790	−0·1440	0·1574
−35	−0·0007	0·109	0·2504	0·1352	−0·1411	−0·2579	0·2763
−30	−0·0022	0·194	0·4403	0·2453	−0·2525	−0·4638	0·4978
−25	−0·0069	0·346	0·7676	0·4312	−0·4538	−0·8421	0·8850
−20	−0·0217	0·616	1·3184	0·7508	−0·8218	−1·5544	1·5726
−18	−0·0343	0·776	1·6264	0·9332	−1.0453	−2·0014	1·9785
−16	−0·0543	0·980	1·9963	1·1558	−1·3326	−2·5931	2·4884
−14	−0·0859	1·238	2·4355	1·4247	−1·7028	−3·3872	3·1275
−12	−0·136	1·569	2·9491	1·7455	−2·1808	−4·4715	3·9263
−10	−0·214	1·999	3·5384	2·1218	−2·7975	−5·9833	4·9193
−8	−0·334	2·565	4·1980	2·5528	−3·5861	−8·1418	6·1389
−6	−0·507	3.279	4·9132	3·0307	−4·5714	−11·0972	7·6021
−4	−0·706	4.036	5·6559	3·5366	−5·7101	−14·2546	9·2467
−2	−0·866	4·667	6·3811	4·0366	−6·7874	−16·4258	10·8240
0	−0·941	5·094	7·0246	4·4782	−7·5267	−17·5512	12·0049
2	−0·953	5·340	7·5228	4·8088	−8·0074	−18·0527	12·8162
4	−0·942	5·465	7·8525	5·0137	−8·0732	−18·2573	13·0869
6	−0·929	5·525	8·0435	5·1233	−8·1386	−18·3361	13·2619
8	−0·922	5·551	8·1417	5·1749	−8·1646	−18·3669	13·3395
10	−0·918	5·562	8·1881	5·1976	−8·1753	−18·3788	13·3729
12	−0·916	5·567	8·2090	5·2071	−8·1792	−18·3834	13·3863
14	−0·916	5·569	8·2179	5·2112	−8·1804	−18·3852	13·3916
16	−0·915	5·570	8·2216	5·2128	−8·1811	−18·3860	13·3939
18	−0·915	5·570	8·2232	5·2135	−8·1813	−18·3863	13·3948
20	−0·915	5·570	8·2238	5·2137	−8·1814	−18·3864	13·3951
∞	−0·915	5·570	8·2242	5·2139	−8·1815	−18·3865	13·3954

TABLE I(*b*)

Characteristics of the Nakagami-Rice phase-interference fading distribution $Y_i(q)$

($Y_i > Y_i(q)$ with probability q; $Y_i(0{\cdot}5) \equiv 0$)

K	$Y_i(0{\cdot}005)$	$Y_i(0{\cdot}02)$	$Y_i(0{\cdot}05)$	$Y_i(0{\cdot}95)$	$Y_i(0{\cdot}98)$	$Y_i(0{\cdot}995)$
dB	dB	dB	dB	dB	dB	dB
−40	0·1568	0·1252	0·1004	− 0·1016	− 0·1270	− 0·1596
−35	0·2768	0·2214	0·1778	− 0·1815	− 0·2272	− 0·2860
−30	0·4862	0·3898	0·3136	− 0·3254	− 0·4082	− 0·5151
−25	0·8460	0·6811	0·5496	− 0·5868	− 0·7391	− 0·9374
−20	1·4486	1·1738	0·9524	− 1·0696	− 1·3572	− 1·7389
−18	1·7840	1·4508	1·1846	− 1·3660	− 1·7416	− 2·2461
−16	2·1856	1·7847	1·4573	− 1·7506	− 2·2463	− 2·9231
−14	2·6605	2·1829	1·7896	− 2·2526	− 2·9156	− 3·8422
−12	3·2136	2·6507	2·1831	− 2·9119	− 3·8143	− 5·1188
−10	3·8453	3·1902	2·6408	− 3·7820	− 5·0372	− 6·9452
− 8	4·5493	3·7975	3·1602	− 4·9287	− 6·7171	− 9·6386
− 6	5·3093	4·4591	3·7313	− 6·4059	− 8·9732	−13·4194
− 4	6·0955	5·1494	4·3315	− 8·1216	−11·5185	−17·1017
− 2	6·8613	5·8252	4·9219	− 9·6278	−13·4690	−19·4073
0	7·5411	6·4248	5·4449	−10·5553	−14·5401	−20·5618
2	8·0697	6·8861	5·8423	−11·0005	−15·0271	−21·0706
4	8·4231	7·1873	6·0956	−11·1876	−15·2273	−21·2774
6	8·6309	7·3588	6·2354	−11·2606	−15·3046	−21·3565
8	8·7394	7·4451	6·3034	−11·2893	−15·3349	−21·3880
10	8·7918	7·4857	6·3341	−11·3005	−15·3466	−21·4000
12	8·8155	7·5031	6·3474	−11·3048	−15·3512	−21·4046
14	8·8258	7·5106	6·3531	−11·3065	−15·3529	−21·4064
16	8·8301	7·5136	6·3552	−11·3072	−15·3537	−21·4072
18	8·8319	7·5149	6·3561	−11·3075	15·3540	−21·4075
20	8·8326	7·5154	6·3565	−11·3076	−15·3541	−21·4076
∞	8·8331	7·5158	6·3567	−11·3077	−15·3542	−21·4077



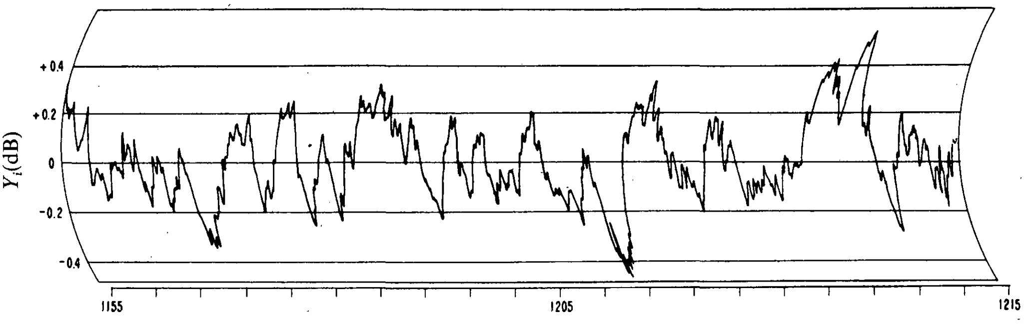
Ratio, K(dB) between the root-sum-square of the amplitudes of the Rayleigh fading component and the amplitude of the steady component of the received signal.

Figure 1

The Nakagami-Rice probability distribution of the instantaneous fading associated with phase interference

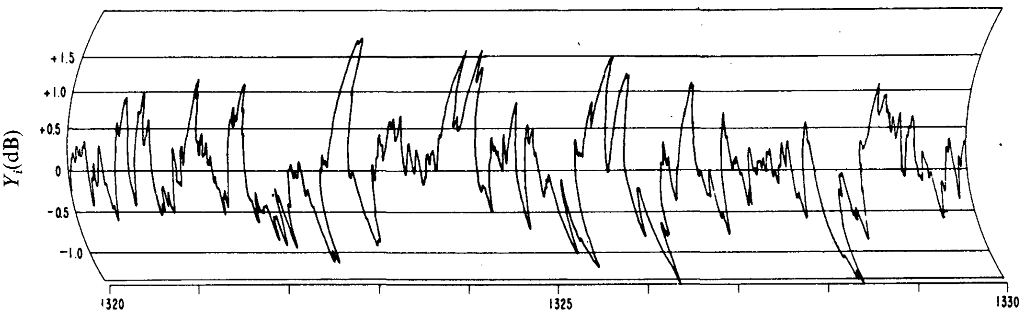
q is the probability that $Y_i \equiv (P_i - P_m) > Y_i(q)$

As K increases without limit, the Nakagami-Rice distribution approaches the Rayleigh distribution



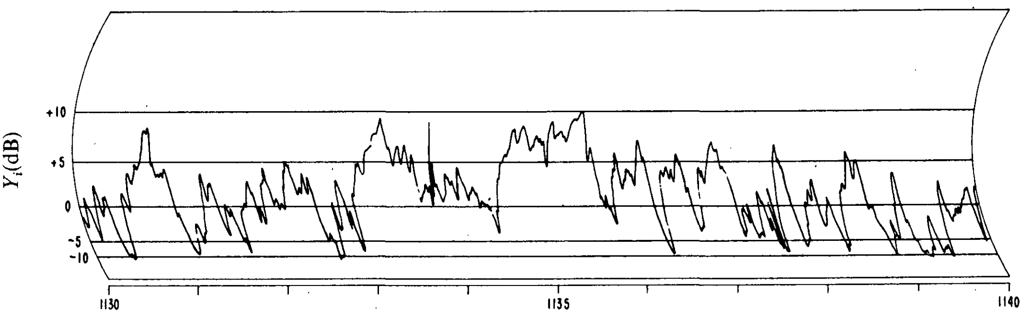
Path length: 70·2 miles; angular distance: $-1{\cdot}33$ mrad
Fading rate: 29 positive crossings in 20 min = 1·45
$Y_i(0{\cdot}1) - Y_i(0{\cdot}9) = 0{\cdot}39$ dB; $K = -32$ dB

(a) *Cheyenne Mountain—Karval, Colorado, 24.VIII.1955*



Path length: 96·6 miles; angular distance: 1·15 mrad
Fading rate: 35 positive crossings in 10 min = 3·5
$Y_i(0{\cdot}1) - Y_i(0{\cdot}9) = 1{\cdot}57$ dB; $K = -20$ dB

(b) *Cheyenne Mountain — Haswell, Colorado, 18.VIII.1954*



Path length: 226·5 miles; angular distance: 28·1 mrad
Fading rate: 32 positive crossings in 10 min = 3·2
$Y_i(0{\cdot}1) - Y_i(0{\cdot}9) = 12{\cdot}6$ dB; assumed value of $K: \gg 20$ dB

(c) *Cheyenne Mountain — Garden City, Kansas, 27.III.1953*

FIGURE 2

Samples of charts for transmissions at 100 MHz from Cheyenne Mountain

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